UNIVERSITY OF CALIFORNIA

Los Angeles

FLATTENED RESOURCE ALLOCATION, HIERARCHY DESIGN AND THE BOUNDARIES OF THE FIRM

A dissertation submitted in partial satisfaction of the

requirements for the degree Doctor of Philosophy

in Economics

by

Szu-Wen Chou

2002

The dissertation of Szu-Wen Chou is approved.

Alberto Bennardo

John G. Riley

Steven Lippman

David K. Levine, Committee Chair

University of California, Los Angeles

2002

Dedicated with love to my parents, wife, and children.

Table of Contents

List of Figures

ACKNOWLEDGEMENTS

I gatefully acknowledge the invaluable discussions with David Levine. I also thank Alberto Bennardo, Hongbin Cai, Walter Cont, Steven Lippman, Joseph Ostroy, John Riley, William Zame, participants of the Theory Proseminar at UCLA, participants of the 11th Jerusalem Summer School in Economic Theory at The Hebrew University of Jerusalem, for helpful commands. All errors are mine.

VITA

August 24, 1969	Born, Kaoshiung, Taiwan
1991- 1993	Research Assistant Graduate School of Business, National Cheng-Kung University, Tainan, Taiwan
1993-1995	Lieutenant (Logistic Officer) Tainan Airport, ROC Air Force, Tainan, Taiwan
1995-1997	Programmer/System Analyst ORIGIN, Kaohsiung, Taiwan

ABSTRACT OF THE DISSERTATION

Flattened Resource Allocation, Hierarchy Design and the Boundaries of the Firm

by

Szu-Wen Chou

Doctor of Philosophy in Economics

University of California, Los Angeles, 2002

Professor David K. Levine, Chair

The general theme of my dissertation is to understand why and how institutions matter in the efficiency of organizations. Empirical results suggest that multi-segment firms underperform stand-alone firms in resource allocations by underinvesting in high-profit segments and overinvesting in low-profit segments - a phenomenon I called flattened resource allocations (henceforth FRA). In the first two chapters I provide a theoretical rationale for FRA and examine its relationship to hierarchy design. The third chapter changes focus to explore the role of the boundaries of the firm in the absence of property rights.

In the first chapter - "Flattened Resource Allocations, Separation of Ownership and Control, and Diversification of the Firm", I present a model of FRA. This model is

more robust than existing FRA models because it abstracts from rent-seeking behavior - a critical but problematic assumption in the existing literature. I show that FRA is a negative externality existing in multi-segment firms but not in stand-alone firms. Also, consistent with the facts from empirical studies, the inefficiency of FRA increases in both the agency cost and diversification of the multi-segment firm. There can be both underinvestment and overinvestment in the total resource available to the firm. These results hold when management ownership is either exogenous or endogenous. In the latter case, the pattern of FRA becomes less clear-cut. More interestingly, the CEO can obtain more than he would receive in a stand-alone firm, explaining why CEOs might prefer managing multi-segment firms. Finally, the total resource available to the firm is lower when management ownership is endogenous than when it is exogenous.

In the second chapter - "Hierarchy Design with Flattened Resource Allocations ", I compare the efficiency of flat and tall hierarchies from the perspective of FRA. I allow the possibility of divisionalization - grouping elementary business segments into a fewer number of divisions and transforming flat hierarchies to tall hierarchies. I characterize the equilibrium of this model and the inefficiency of FRA is shown to be decreasing in management ownership. Most importantly, FRA is aggravated by divisionalization, implying tall hierarchies are necessarily less efficient than flat hierarchies. This result suggests that U-form (undivisionalized) organizations, represented by flat hierarchies, can actually outperform M-form (divisionalized) organizations, represented by tall hierarchies, in resource allocations – a result opposite to

Williamson’s ”M-form hypothesis.”

In the third chapter - “The Boundaries of the Firms in the Absence of Property Rights” it is shown that the boundaries of the firm can continue to matter in an environment where there is no physical asset and hence property rights do not play a role. I assume that the boundaries of the firm work as a form of information barrier: once parties enter the boundaries of the firm they work together behind ’frosted-glass windows’ and hence their outside options get averaged out. I show that mergers can induce first-best investment. This new view provides a rationale for the large observed investment in adjusting firm boundaries in industries where physical assets are of minimal importance.

1 Flattened Resource Allocations, Separation of Ownership and Control, and Diversification of the Firm

1.1 Introduction

In most market economies, productive resources such as labor and capital are allocated to the firm through *the external* (meaning outside of the boundaries of the firm) *resource market* by some price mechanism; the firm in turn allocate the resources it obtains to each elementary business segment inside the firm through *the internal resource market* by its central office or CEO. The common goal of these markets is to assign resources to high-yield uses. While the external resource market is free of human discretion but prone to failure when there is asymmetric information, the internal resource market has a lower information cost but is subject to agency problems. The relative efficiency of these two different markets is one of the major factor in deciding the optimal boundaries of the firm – whether to organize a given group of business segments as several *stand-alone firms* or as one *multi-segment firm*. For a multi-segment firm to operate as a internal resource market, it is necessary that the segments inside the firm are somewhat different from each other, i.e. there must be some degree of *diversification*, so that it matters when the firm's CEO redirect resources from one segment to another segment.

In the past, possibly due to the less sophisticated technologies used in the external resource market, it was believed that the information problem is the major issue and hence resource allocation would be better done inside the boundaries of the firm. One of the most representative arguments is proposed by Williamson [33], who argues that the internal capital market should be more efficient than the external capital market because the internal market has lower information cost, better ability to fine-tune allocations, and lower intervention cost. This point of view is so influential that it still inspires some of the most recent studies. For example, Stein [29] suggests that the internal capital market should be more efficient because the CEO of multi-segment firms can have the necessary information and incentive in *picking winners* among segments inside the firm, resulting better capital allocations, compared to the external capital market where all segments are treated equally for lack of information. Similarly, Inderst and Müller [17] suppose that liquidity spillovers from high- to low-return segments can help in financing low-return but positive NPV segments and hence enhance firm value.

However, as information technologies advance, the internal resource market has been losing its advantage. Consequently, some of the potential problems of the internal resource market, especially the internal *capital* market, start to surface. One recent important observation is that many diversified conglomerates – firms with many unrelated segments and hence major candidates for the internal resource market – perform poorly relative to specialized firms with less unrelated segments. For example, Lang and Stulz [20] and Berger and Ofek [3] find that diversified firms are valued at a discount relative to specialized firms in the U.S. stock market, suggesting that the internal capital market can be more costly than the external capital market. Additionally, Scharfstein [26] reports that 68% of his sample of diversified conglomerates existing in 1979 either sold off unrelated segments or got acquired or liquidated by 1994. As for what might have gone wrong in these multi-segment firms, he finds that, compared to their stand-alone industrial peers, segments in high-profit industries tend to be underinvested in, whereas segments in low-profit industries tend to be overinvested in – a peculiar pattern of misallocation that we called *flattened resource allocations* (henceforth FRA). In addition, the degree of FRA decreases in CEO ownership and increases in the diversify (Rajan, Servaes and Zingales [24]) of the firm. While these empirical results are still under debate[1], they do shift the theoretical attention from the better studied, bright side of the internal resource market to the less explored, dark side of it.

The theoretical underpinning of FRA turns out to be quite challenging. While it is just the standard Jensen-Merkling type of result that managers tend to be *extravagant* with investment because they enjoy the full benefit of the perks that come with investment but only bear a portion of the cost, it is hard to rationalize why managers want to *flatten* resource allocations that is to be simultaneously extravagant towards some segments but *stingy* towards other segments. Another related, interesting question is that if diversification really destroys value, why does it happen to begin with? Morck, Shleifer and Vishny [23] suggest that managerial objectives might be the driving force.

[1] For example, Whited [32] argues that these results can be subject to measurement errors in the proxy for profitability – Tobin's q.

In this paper, we are interested in finding out whether managing a diversified multi-segment firm allows the CEO to gain more rent than managing a stand-alone firm does.

The followings are our research questions: 1.) Why and how do FRA happen in multi-segment firms but not in stand-alone firms? 2.) Why and how do FRA depend on managerial ownership and diversity of the firm? 3.) How does the total resource available to a certain group of business segments change with the boundaries of the firm? 4.) What are the effects of allowing stock-based compensation on FRA, and what new effects will it generate in the relationship between the owner and the CEO of a firm? and 5.) What in the internal resource market, if anything, gives CEOs the incentive to create multi-segment firms by acquiring unrelated segments?

In this paper, we consider two segments, different in profitability, that can be separated as two stand-alone firms or integrated as one multi-segment firm. In both cases, a firm constitutes a two-tier principal-agent problem with its owner as the principal and its CEO as the agent at the upper tier and the CEO in turn as the principal and one or two segment manager(s) (depending on whether the firm is multi-segment or stand-alone) as the agent(s) at the lower tier. With the cash and production resource provided by the owner, the CEO decides (a) how to distribute the cash among himself and his segment manager(s) to satisfy her(their) participation constraint(s) and (b) how to distribute the resource to his segment manager(s) so that the resource can be converted into revenue. The owners and CEOs only care about their wealth, whereas the segment managers care about wealth and a private benefit of *empire building*, modeled as an additive part of each segment manager's utility function that is increasing and concave in the resource allocated to the segment. In each of the stand-alone firm, the CEO will simply pass on all of the resource that he received from the owner to his segment manager because he controls only one segment and cannot use the resource in any other ways. As a result, the resource allocations will be first-best, and by construction the high-profit segment will be allocated more resource. This implies that the manager of the high-profit segment will have a lower marginal utility of resource than manager of the low-profit segment.

On the other hand, in the multi-segment firm the CEO can distribute the resource to the two segments in any way he wants. If the CEO wants to reduce cash payment, he can do so by the following deviation from the first best: redirecting some of the resource from the segment manager who has low marginal utility of resource (the high-profit segment) to the segment manager who has high marginal utility of resource (the low-profit segment) while still keeping the segment managers receiving their reservation utilities. It tuns out that this inefficient deviation – FRA – is actually preferred by the CEO because he can enjoy the full benefit of cash saving but, as a agent of the owner, does not bear the full cost of resource misallocation.

Concentrating on the multi-segment firm, we also show that there could be *both* underinvestment *and* overinvestment in the total resource available to the firm as the owner's optimal responses to the CEO's resource misallocation. The possibility of investors' *over*investment in equilibrium is one of the important feature that distinguishes this paper from most of the corporate finance literature, where it is always some variety of resource (especially capital) *under*investment (for example debt contracts) derived as investors' optimal response to managers' abuse of the investors' resource. Furthermore, we show that the efficiency loss of FRA is increasing in the degree of separation of ownership from control and diversity of the firm. All these results hold quantitatively the same irrespective of whether CEO ownership is exogenous or endogenous. However, when CEO ownership is endogenous, two additional effects obtain: First, the total resource available to the multi-segment firm becomes lower. This reduced provision of resource by the owner makes the pattern of FRA more subtle because underinvestment in both segments becomes possible. Second, the CEO of the multi-segment firm can receive utility higher than what he would receive in the stand-alone firms. This provides a rationale for why CEOs want to acquire unrelated segments and create a diversified multi-segment firm to begin with.

Most of the existing theoretical literature on FRA follows the *rent-seeking approach* broadly defined as *models assuming there exist certain socially wasteful technologies that allow managers to seek personal rent.* For example, Rajan, Servaes and Zingales [24] (henceforth RSZ) assume that the managers of two different segments can use the resource provided by the CEO to make wasteful "defensive investment" instead of efficient investment so that their surplus can be protected from being poached by another segment manager. In their model, the CEO, who is also assumed to be the owner, will *efficiently* tilt resource allocations towards the low-profit segment because that will discourage the manager of the *high*-profit segment to make defensive investment which in turn induces the manager of the low-profit segment also to make efficient investment.

In another rent-seeking model, Scharfstein and Stein [27] (henceforth SS) assume that the managers of two different segments, instead of spending all of their effort in production in this period, can choose to spend some of their effort in unproductive rent-seeking activities such as "resume polishing" that will improve their outside options in the next period. On one hand, the manager of the low-profit segment has a lower opportunity cost to spend her effort in rent seeking because her effort is assumed to be less productive. On the other hand, the payoff of the manager of the low-profit segment in the next period will be lower than the manager of the high-profit segment because the return on the newly invested capital in the low-profit segment is also *assumed* to be lower. This implies the manager of the *low*-profit segment should care more about her outside option and hence has not only a lower cost but also a higher benefit in rent seeking. To discourage the manager of the low-profit segment from rent seeking the CEO need to raise her payoff inside the firm by paying her a positive wage or tilting resource allocations towards her segment. Resource allocations are then flattened because the CEO actually rather "pays" the manager of the low-profit segment partially by tilting the resource allocations than completely by a positive wage because the CEO enjoys the full benefit of wage saving but does not bear the full cost of resource misallocation. It is important to note that the two papers discussed above employ *opposite* assumptions about which segment manager has stronger tendency to rent seek: RSZ assume the manager of the *high*-profit segment is more likely to rent seek, whereas SS assume the manager of the *low*-profit segment is more likely to rent seek. While both settings seem plausible, this modeling discretion turns out to have a substantial impact on the results, as discussed in the next paragraph.

The rent-seeking models share two major defects. First, and most importantly, their results are very sensitive to the discretional designs of the rent-seeking games used in the models. In other words, a slight and reasonable change of the model can *overturn the results*. For example, in SS if one adds the assumption that *segment managers' outside options are positively related to the segment's productivity*, so that it is the manager of the *high*-profit segment whose outside option is more likely to bind, completely opposite results obtain.[2] The assumption added above is by no means implausible. In fact, as we point out above, it is consistent with model of RSZ where the manager of the *high*-profit segment is more likely to rent seek. In the current paper, we avoid this problem by simply abstracting from any rent-seeking behavior which, as it turns out, is totally unnecessary for modeling FRA.

The second important common flaw of the rent-seeking models is that the total resource available to the firm is exogenous. This leads to the following two difficulties. First, these models cannot explain how the total resource available to the firm

[2] Similar problems exist in RSZ.

change as a response to the CEO's resource misallocation. Probably due to the literature's predominant results on investors' underinvestment as a response to manager's exploitation, SS allege, without formal analysis, that FRA will simply lead to *under*investment in total resource available to the firm. We clarify this myth by formally showing that not only underinvestment but also *over*investment in the total resource available to the firm can be an equilibrium outcome under the problem of FRA. Second, these existing models cannot explain why the ultimate resource provider (the owner), if allowed by the modeler, will not decide to substantially underinvest or overinvest in the firm to the point that *all* segments are underinvested or overinvested. These two scenarios are anomalies that need to be ruled out for any model to fully explain FRA. We overcome these two difficulties by endogenizing the total resource available to the firm.[3]

In addition to fixing the two major defects discussed above, we improve upon the existing literature by endogenizing managerial ownership. This not only allows us to examine the effects of stock-based compensation on FRA but also to provide conditions under which a CEO will prefer managing a multi-segment firm to a stand-alone firm.

The current paper has a different focus from the related capital budgeting literature (see for example Harris and Raviv [15][16] and Bernardo, Cai and Luo[4]). While

[3] To ensure finite benchmark allocations and private increasing in resource, we also deviate from some technical formulations of SS (where segment profit functions are increasing and private benefit is modeled as utility derived from profit) and adopt the formulations of Harris and Raviv [15][16] (where segment profit functions are single-peak and private benefit is modeled as utility derived from resource). These two formulations are, in our opinion, equally plausible.

it also deals with the internal financing of the firm in general, the capital budgeting literature focuses on understanding the design of setting a budgeting limit that can be lifted under some conditions. The goal of this paper is to understand why resource allocations change with firm boundaries, which do not play a role in capital budgeting literature.

The rest of the paper is organized as follows: We first restrict our attention in section 2.2. to the case of exogenous manager ownership. In section 2.3., we endogenize the ownership of the managers. Discussion and conclusion are presented in section 3.

1.2 The Model

1.2.1 Assumptions and Definitions

There are two different business segments, indexed by i where $i \in \{1, 2\}$. The revenue function of each segment is $V_i(k_i)$, a function of resource input k_i. Let segment 1 be the high-profit segment in the sense that it is in the industry with relatively *good* investment opportunity[4], compared to segment 2. Technically, we assume the following:

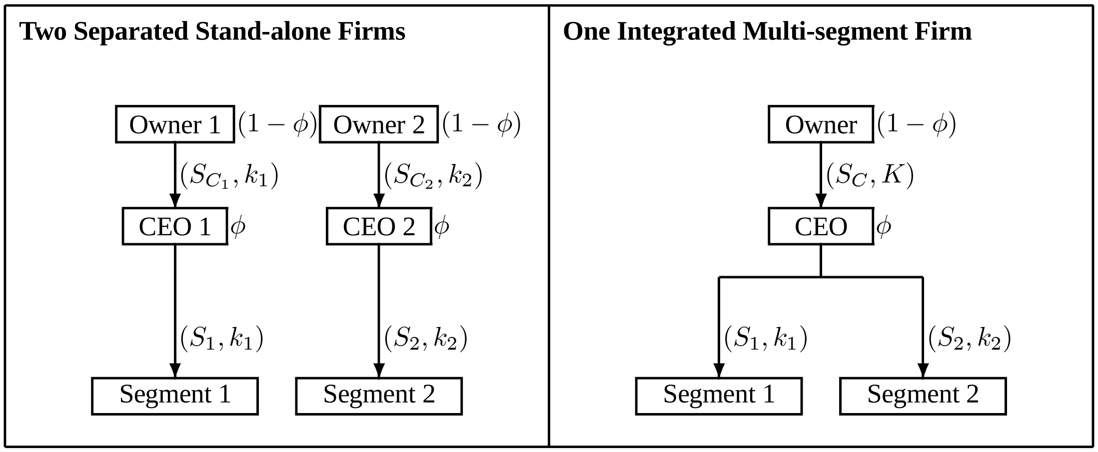
Assumption 1: $V_i : \mathbf{R} \to \mathbf{R}$, $i \in \{1, 2\}$, *is a continuously differentiable strictly concave function which satisfies*

$$V_i(0) = 0,\ V_2'(k) < V_1'(k), \text{ and } V_i''(k) > -\infty. \tag{1}$$

As shown in Figure 1., the two segments can be organized as either two separated stand-alone firms or one multi-segment firm. In the former case, we refer the firm

[4] One of the measures of profitability of an industry used in empirical works is Tobin's Q.

Figure 1: Separation and Integration



that segment i belongs to as firm i. Each firm i consists one *owner*, one *CEO*, and one *segment manager*. In the latter case, the firm consists one owner, one CEO, and two segment managers. The profit of firm i is $V_i(k_i) - k_i$, denoted as $\Phi_i(k_i)$. The profit of the multi-segment firm is the sum of the profit of the two segments, which is $V_1(k_1) + V_2(k_2) - k_1 - k_2$, denoted as $\Phi(k_1, k_2)$.

Each firm (stand-alone or multi-segment) constitutes a two-tier principal-agent problem with its owner as the principal and its CEO as the agent at the upper tier and the CEO as the principal and one or two segment manager(s) as the agent(s) at the lower tier. For simplicity, we assume that any other way of organization will be very costly. For example, a firm cannot operate without a CEO because it takes the unique expertise of the CEO to supervise the segment manager(s).

All agents need to be paid a non-negative amount of cash to satisfy their participation constraints. The "cash" is broadly defined to include *all means of rewarding a employee*, for example salary, employee benefit, and even perks. Cash cannot be used as resource to be converted into revenue. Each owner has a unlimited endowment of resource and cash, whereas no CEO or segment manager has any endowment. This implies that the resource and cash a CEO provides to his segment manager(s) necessarily come from what is provided to him by his owner.

Since part of our goal is to replicate the results of SS, without invoking rent-seeking behaviors, we follow SS in the following three assumptions (Assumptions 2, 3, and 4). While SS use them to model the multi-segment firm only, we apply them to *both* the stand-alone firms and the multi-segment firm so that the results in both types of firms are comparable.

Assumption 2: *The principals are in charge of both allocating resource to and retaining their direct agents.*

Specifically, in each firm the CEO is in charge of allocating resource to his segment manager(s) and paying his segment manager(s) a non-negative amount of cash to satisfy her(their) participation constraint(s); the owner is responsible for allocating resource to her CEO and paying her CEO a non-negative amount of cash to satisfy his participation constraint.

Assumption 3: *As long as the participation constraint(s) of his segment manager(s) is(are) satisfied, the CEO can divert any cash received from his owner but is* *not paid out to his segment managers.*

Denote the cash that the manager of segment i gets paid as S_i. Denote also the the gross amount of cash that the CEO gets paid as S if he is the CEO of the multi-segment firm or as S_{C_i} if he is the CEO of the stand-alone firm i. Under Assumption 3, S or S_{C_i} really should be viewed as a ”personnel budget”[5] for the firm which consists of two parts: the salary of the manager, S_i, and the salary of the CEO, $S_{C_i} - S_i$ or $S - \sum_i S_i$.

Assumption 4: *In each firm, the resource and cash provided to each segment manager by its CEOs are not contractible to its owner.*

Under this assumption, the only things the owner can control are the total resource and cash provided to the firm as a whole. Once the owner parts with her resource and cash, she has no effective control over how they get distributed to each segment of the firm. This assumption reflects the well-known difficulty in attributing a firm's resource usage to individual segments in accurate accounting terms especially when the resource is shared among several segments.

While Assumptions 2, 3, and 4 are important in creating the scope for agency problems in general, they are not sufficient to entail FRA. In order to equip the CEO with the special incentive to fatten the resource allocations, SS assume that segment managers can rent-seek and derive private benefit from resource. We find that only

[5] This is also called ”operating budget” in Scharfstein and Stein [27]. The assumption that the CEOs can divert the personnel budget that is not spend on their subordinates reflects the prevalent channels of side payment in practice between managers and their subordinates such as personal favor.

the latter ingredient is necessary. Specifically, we assume that while the owner and the CEO care only about their wealth (the total value of cash and shares they own), the utility functions of the manager of each segment i is the sum of her wealth *and* a private benefit of *empire building* $b(k_i)$ that satisfies the following assumption:

Assumption 5: $b : \mathbf{R} \rightarrow \mathbf{R}$, *is a increasing, strictly concave, and continuously differentiable function which satisfies* $b''(k) > -\infty$, $\lim_{k_i \rightarrow 0+} b'(k_i) = \infty$.

To rule out non-instructive corner solutions where the agents receiving zero net cash, we also make the following simplifying assumption:

Assumption 6: *The reservation values the agents (the CEO(s) and the segment managers) are high enough (in particular strictly greater then zero) so that their participation constraints cannot be satisfied without receiving a positive amount of net cash or stock-based compensation.*

There are two potential benchmarks for segment resource allocations. Which of them is revelant depends on the choise of the stand-alone firms. We assume that both of them are unique and defined as follows:

Definition 1 *The **first-best** resource allocation for segment i is k_i^* such that*

$$k_i^* = \arg\max_{k_i}[V_i(k_i) + b(k_i) - k_i]. \tag{2}$$

Definition 2 *The **production profit maximizing** resource allocation for segment i is k_i^- such that*

$$k_i^- = \arg\max_{k_i}[V_i(k_i) - k_i].$$

Given that both k_i^* and k_i^- are unique, inequality (1) combined with the monotonicity and concavity of the private benefit function $b(.)$ implies that in both benchmark cases the resource allocated to the high-profit segment (segment 1) should be higher than the resource allocated to the low-profit segment (segment 2) i.e.

$$k_1^* > k_2^* \text{ and } k_1^- > k_2^-.$$

Also it is obvious that $k_i^* > k_i^-$.

The following two definitions help characterize inefficiency.

Definition 3 *There exists diversification discount if the value of the multi-segment firm is lower than the sum of the values of the two stand-alone firms.*

Definition 4 *There is overinvestment in the total resource available to the firm if and only if $k_1 + k_2 > k_1^* + k_2^*$; there is underinvestment in the total resource available to the firm if and only if $k_1 + k_2 < k_1^* + k_2^*$.*

Finally, the owner, the CEO, and the segment managers of each firm are all potential shareholders of the firm. The residual income of the firm is distributed to its shareholders according to their fraction of shares.

1.2.2 Exogenous Management Ownership

In this section, we assume that the shares of each firm owned by its CEO is exogenously given. This assumption reflects many real-life cases where there exist some difficulties in using stock-based compensation. For example, some shareholder might be concerned about maintaining their status as majority shareholders, or some CEOs might have liquidity needs which cannot be satisfied by getting shares. Without loss of generosity, we can assume that the segment managers does not own any shares since as it turns out what matters is the *total* shares owned by each firm's CEO and segment manager(s). Therefore, we can interpret the shares owned by the CEO as employee (excluding the owner) or managerial ownership.

Optimal Resource Allocations in the Stand-alone Firm In this sub-section we analyze resource allocations in a representative stand-alone firm – firm i. Denote the portion of the shares owned by the CEO as ϕ_{C_i}. According to our setup above, the utility function of the owner should be

$$(1 - \phi_{C_i})\Phi_i(k_i) - S_{C_i},$$

and utility function of the CEO should be

$$\phi_{C_i}\Phi_i(k_i) + S_{C_i} - S_i.$$

Therefore, the two-tier principal-agent problem can be written as finding (S_{C_i}, k_i) such that

$$(S_{C_i}, k_i) = \arg\max_{(\widetilde{S_{C_i}}, \widetilde{k_i})} -\widetilde{S_{C_i}} + (1 - \phi_{C_i})\Phi_i(\widetilde{k_i}) \tag{3}$$

subject to

$$\phi_{C_i}\Phi_i(\widetilde{k_i}) + \widetilde{S_{C_i}} - S_i \geq \overline{u}_C \tag{4}$$

$$S_i = \arg\max_{\widetilde{S}_i} \phi_{C_i}\Phi_i(\widetilde{k_i}) + \widetilde{S_{C_i}} - \widetilde{S}_i$$

subject to

$$\widetilde{S}_i \leq \widetilde{S_{C_i}}$$

$$\widetilde{S}_i + b(\widetilde{k_i}) \geq \overline{u}_{M_i}, \tag{5}$$

with $\overline{u}_C$ and $\overline{u}_{M_i}$ representing the reservation values of the CEO and the manager of segment i respectively and all decision variables nonnegative.

Since both equation (5) and equation (4) will hold as equality in equilibrium by Assumption 6, substituting them into equation (3) gives us

$$\max_{\widetilde{k}_i}[V_i(\widetilde{k}_i) + b(\widetilde{k}_i) - \widetilde{k}_i - \overline{u}_{C_i} - \overline{u}_{M_i}]. \tag{6}$$

Also since the first-order condition for equation (6) is the same as for equation (2), the first-best resource allocations is achieved in this case. Define variables with superscript "SX" as their respective solutions in the stand-alone firm when the CEO's ownership is exogenous. Then the solution to this case is

$$\begin{aligned} & (S_{C_1}^{SX}, S_{C_2}^{SX}, S_1^{SX}, S_2^{SX}, k_1^{SX}, k_2^{SX}) \\ = \quad & (\overline{u}_{C_1} + S_1^{SX}, \overline{u}_{C_2} + S_2^{SX}, \overline{u}_{M_1} - b(k_1), \overline{u}_{M_2} - b(k_2), k_1^*, k_2^*). \end{aligned}$$

To conclude this sub-section, even though we assume that the segmental resource allocations are not contractible, the potential FRA problem will not happen in the stand-alone firms simply because in each stand-alone firms there is only one segment to which the CEO can allocate resource, and hence the CEO has no way to misallocate it.

Optimal Resource Allocations in the Multi-segment Firm In this sub-section we will analyze resource allocations in the multi-segment firm. Denote the portion of the shares owned by the CEO as ϕ_C. According to our setup, the utility function of the owner is

$$(1 - \phi_C)\Phi(k_1, k_2) - S,$$

while utility function of the CEO is

$$\phi_C \Phi(k_1, k_2) + S - (S_1 + S_2)\,.$$

Therefore, the two-tier principal-agent problem is to find (S, K) such that

$$(S, K) = \arg\max_{(\widetilde{S}, \widetilde{K})} -\widetilde{S} + (1-\phi)\Phi(k_1, k_2)$$

subject to

$$\widetilde{S} + \phi\Phi(k_1, k_2) - (S_1 + S_2) \geq \overline{u}_C \tag{7}$$

$$(S_1, S_2, k_1, k_2) = \arg\max_{(\widetilde{S_1}, \widetilde{S_2}, \widetilde{k_1}, \widetilde{k_2})} \{\widetilde{S} + \phi\Phi(\widetilde{k_1}, \widetilde{k_2}) - (\widetilde{S_1} + \widetilde{S_2})\} \tag{8}$$

subject to

$$\widetilde{k_1} + \widetilde{k_2} \leq \widetilde{K}$$

$$\widetilde{S_1} + \widetilde{S_2} \leq \widetilde{S} \tag{9}$$

$$\widetilde{S_i} + b(\widetilde{k_i}) \geq \overline{u}_{M_i} \text{ for } i \in \{1, 2\}, \tag{10}$$

with $\overline{u}_C$ and $\overline{u}_{M_i}$ representing the reservation values of the CEO and the manager of segment i respectively and all decision variables nonnegative.

Define variables with superscript "MX" as their respective solutions in the multi-segment firm when the CEO's ownership is exogenous. The following result characterizes a *partial* solution that highlights the source of inefficiency.

Proposition 1 *(Externality in Internal Resource Market) Given the owner's decision is (S, K), which satisfies equations (7) and (9), the solution to the CEO's problem is*

$$(S_1^{MX}, S_2^{MX}, k_1^{MX}, k_2^{MX}) = [\overline{u}_{M_1} - b(k_1), \overline{u}_{M_2} - b(k_2), k_1, K - k_1], \tag{11}$$

where k_1 satisfies

$$\phi[V_1'(k_1) - V_2'(K - k_1)] + b'(k_1) - b'(K - k_1) = 0. \tag{12}$$

Proof. All proofs of this chapter are in Appendix A. ■

Equation (12) is a instructive representation of the working of an internal resource market. Imagine there are three "commodities" in this "market": *resource for segment 1*, *resource for segment 2,* and *cash*. The CEO "produces" the resource for segments 1 and 2 using the total resource available to the firm as input. The segment managers can "buy" ("sell") resource from (to) the CEO by reducing (increasing) their cash received from the CEO. This market differs from ordinary markets in that it is a *sub-market* in the sense that it excludes one of the members of the "society" – the owner – from trading in it. As a result, this sub-market operates and reaches a *sub-social* optimal allocation by equating the *social* marginal benefit, $b'(K - k_1) - b'(k_1)$, with the *sub-social* marginal cost, $\phi[V_1'(k_1) - V_2'(K - k_1)]$, which is strictly lower than the *social* marginal cost, $V_1'(k_1) - V_2'(K - k_1)$. It is exactly this externality that causes inefficiency in the multi-segment firm.

One might notice that equation (12) looks similar to the central first-order condition of SS. However, it is important to note that the purpose of this paper is *not* to challenge SS's results. In fact, a portion of our goal is to *replicate* SS's results in a more robust setting free of rent seeking modeling. This is accomplished by deriving equation (12) that plays a important role in the following part of the analysis that is *absent* in SS. In addition, there are some problems in simply relabeling the central first-order condition of SS into equation (12). For example, SS's analogous parameter for our ϕ represents a private benefit parameter which is supposed to be very small so that private benefits can be neglected in the definition of benchmark allocations. Otherwise, their benchmark allocations cannot represent the first-best allocations. Therefore, their model cannot be used to address the effect of managerial ownership by simply reinterpreting the private benefit parameter as ownership because managerial ownership can be substantial.

Since the segment managers are always getting their reservation values in either a stand-alone or multi-segment firm, any change on the segment's resource allocation should come with an offsetting change in the segment-manager's salary. The following corollary characterizes this observation.

Corollary 2 *If the segment managers have the same reservation value in either the stand-alone or multi-segment firm, then in the multi-segment firm the resource allocations are flattened if and only if the wages of the segment managers are flattened, i.e.*

$$S_1^{MX} > S_1^{SX} \text{ and } S_2^{MX} < S_2^{SX}.$$

The next result is a comparative-static analysis on the behavior of the CEO, equation (12).

Proposition 3 *(a) The resource allocations to the two segments will move apart from each other as managerial ownership increases, i.e.*

$$\frac{\partial k_1^{MX}}{\partial \phi} > 0, \frac{\partial k_2^{MX}}{\partial \phi} < 0.$$

(b) The resource allocation to one of the segment will increases in the resource allocation to the other, i.e.

$$0 < \frac{\partial k_2^{MX}}{\partial k_1^{MX}} < \infty.$$

In the next result, we examine how changes in total resource available to the firm provided by the owner get distributed to each segment.

Proposition 4 *(Socialism in the Internal Resource Market)*

$$0 < \frac{\partial k_1^{MX}}{\partial K^{MX}} < 1 \text{ and } 0 < \frac{\partial k_2^{MX}}{\partial K^{MX}} < 1. \tag{13}$$

Proposition 4 shows that whenever the owner increases or decreases the total resource available to the firm, the allocations to *both* segments will move *in the same direction*. This is referred as "socialism" in the language of Bolton and Scharfstein [7] because whenever the owner wants to increase (decrease) her investment in the firm because, for example, she wants to increase (decrease) the investment in a certain segment, this segment is not going to obtain 100% of the increase (decrease) since this increase will be shared by the other segment. This result is consistent with the empirical finding of Shin and Stulz [28] that one segment's investment depends on the cash flow of the firm's other segments.

In the following result, we present the equilibrium solution to the whole two-tier principal-agent problem.

Proposition 5 *The equilibrium solution of the two-tier principal-agent problem is*

$$\begin{array}{c} (S^{MX}, K^{MX}, S_1^{MX}, S_2^{MX}, k_1^{MX}, k_2^{MX}) = \\ \{\overline{u}_C - \phi\Phi\left[k_1, f(\phi, k_1)\right] + S_1^{MX} + S_2^{MX}, k_1 + f(\phi, k_1), \\ \overline{u}_{M_1} - b(k_1), \overline{u}_{M_2} - b\left[f(\phi, k_1)\right], k_1, f(\phi, k_1)\} \end{array},$$

where k_1 satisfies

$$V_1'(k_1) + b'(k_1) - 1 + \{V_2'[f(\phi, k_1)] + b'[f(\phi, k_1)] - 1\} \frac{\partial f(\phi, k_1)}{\partial k_1} = 0. \qquad (14)$$

In this case, the CEO is receiving exactly his reservation utility.

The first-order condition for the owner's problem, equation (14), consists of two components,

$$V_1'(k_1) + b'(k_1) - 1$$

and

$$\{V_2'[f(\phi, k_1)] + b'[f(\phi, k_1)] - 1\} \, \partial f(\phi, k_1)/\partial k_1,$$

representing the marginal benefit and cost of increasing k_1^M, respectively. The term $\partial f(\phi, k_1)/\partial k_1$ can be viewed as a weighting factor depending on how the CEO distributes the change in the total resource available to the firm between the two segments.

The next result follows directly from the fact that the CEO is always receiving exactly his reservation utility either in the stand-alone firm or the multi-segment firm. We list it here to facilitate comparison with Corollary 5 in the case of endogenous managerial ownership.

Corollary 6 *When managerial ownership is exogenous, the CEO is indifferent between being the CEO of the stand-alone firm and the multi-segment firm.*

The next proposition is one of our main results – flattened resource allocations exist in the multi-segment firm as long as there is separation of ownership and control.

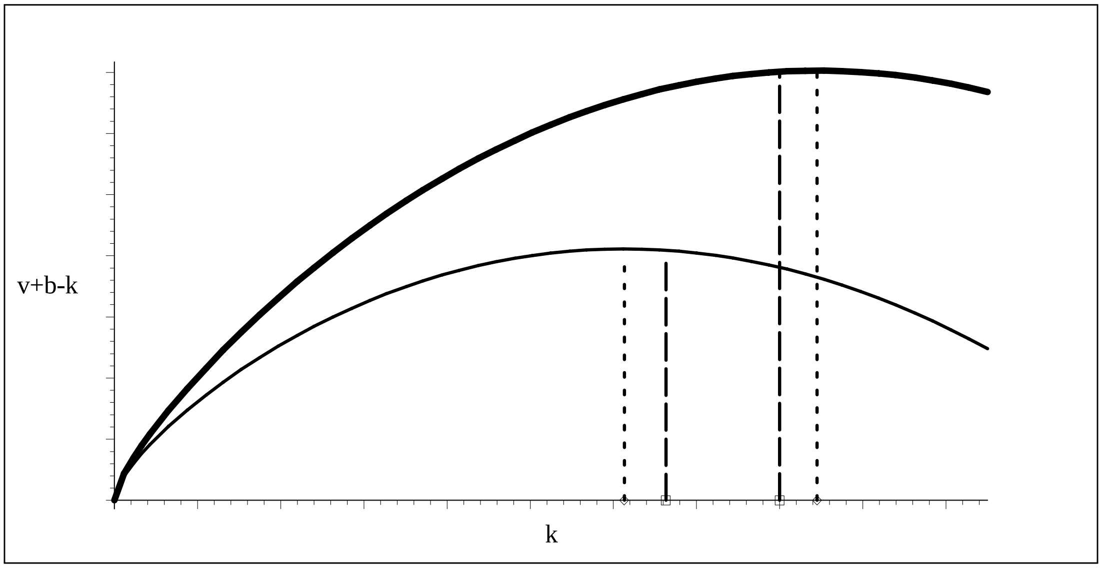


Figure 2: Flattened Resource Allocations ($k_1^M < k_1^*$ and $k_2^M > k_2^*$.)

Proposition 7 (FRA and Diversification Discount) *As long as the CEO is not the sole owner ($\phi < 1$) of the firm, then we have*

(a) the resource allocations in the multi-segment firm will be flattened, i.e.

$$k_1^{MX} < k_1^* \text{ and } k_2^{MX} > k_2^*.$$

(b) There exists diversification discount.

The result is also presented in Figure 2. where one can see that the multi-segment firm (as indicated by the dashed lines) does not discriminate the two segments as adequately as the stand-alone firms do (as indicated by the dotted lines), in terms of resource allocations.

Since the resource allocations are first-best in the stand-alone firms but flattened in the multi-segment firm, the following result obtains directly.

Corollary 8 *It is more efficient for the two segment to be separated as two stand-alone firms than integrated as one multi-segment firm.*

This result explains why the majority of those diversified conglomerates in Scharfstein's [26] sample are broken up and also why acquiring firms will experience negative returns when they announce unrelated acquisitions (Morck, Shleifer, and Vishny [23]). Note that in this paper we do not intend to incorporate any benefit of integration because it has been well-studied and generally can be considered independent of the current paper.

To set a benchmark for the analysis of overinvestment or underinvestment in the total resource available to the firm, we make the following definition:

Definition 5 *Define $\widehat{k_1}$ as the resource that segment 1 gets allocated when the investment in the total resource available to the multi-segment firm is first-best, i.e.* $\widehat{k_1} + f(\phi, \widehat{k_1}) = k_1^* + k_2^*$.

The next result provides conditions of overinvestment or underinvestment in the total resource available to the firm.

Proposition 9 *(The total resource available to the multi-segment firm) There will be overinvestment in the total resource available to the multi-segment firm if*

$$V_1'(\widehat{k_1}) + b'(\widehat{k_1}) - 1 + \left\{ V_2' \left[f(\phi, \widehat{k_1}) \right] + b' \left[f(\phi, \widehat{k_1}) \right] - 1 \right\} \frac{\partial f(\phi, \widehat{k_1})}{\partial k_1} > 0; \quad (15)$$

and there will be underinvestment in the total resource available to the multi-segment firm if

$$V_1'(\widehat{k_1}) + b'(\widehat{k_1}) - 1 + \left\{ V_2' \left[f(\phi, \widehat{k_1}) \right] + b' \left[f(\phi, \widehat{k_1}) \right] - 1 \right\} \frac{\partial f(\phi, \widehat{k_1})}{\partial k_1} < 0. \quad (16)$$

One interesting aspect of Proposition 9 is to discover the role of *over*investment, in the total resource available to the firm, in alleviating managers' agency problem[6]. Pi-

[6] Note that we are restricting our attention to capital misallocation caused by *moral hazard*. In the case of *adverse selection*, there are some results where overinvestment and underinvestment co-exist to induce truth-telling, for example, Antel and Eppen [2] and Harris and Raviv [15].

oneered by Jensen and Mercklin [19], a vast literature on corporate finance has generally found that the optimal defence of investors against managers' abuse of investors' resource is always some forms of *under*investment (investment less than the first-best level). For example, Hart and Moore [14] maintain that debt contracts can refrain managers from diverting investors' resource, and underinvestment optimally arises in the form of setting repayments too high that can lead to inefficient liquidation. The disproportional emphasis on underinvestment are due to the common driving force underlying most agency problems that have been studied – managers are always *extravagant* with investors' resource. In fact, it is hard to imagine why investors would ever want *over*invest if managers simply want to spend as much of the investors' resource as possible. However, investors' overinvestment happens naturally in our setting because the problem at hand is really two-fold: the CEO is simultaneously (a) extravagant towards the low-profit segment and (b) *stingy* towards the high-profit segment. Therefore, when (a) is more costly than (b), it could be optimal for the owner (investor) to underinvest and try to keep some resource out of the reach of the CEO; when (b) is more of a problem than (a), it could be optimal for the owner to overinvest and give the CEO some additional resource to play with. What distinguish this paper from the standard Jensen-Mercklin type of results is exactly this two-fold feature. It is important to note that the two-fold feature of our model does not come directly from any non-standard property of the CEO's utility function.[7] In fact, *the*

[7] For example, we do not assume that the CEO somehow prefer "fair" allocations between the two segments that could directly lead to FRA.

CEO himself does not enjoy any private benefit from investment. It is the combination of (i) the CEO's simultaneous control over multiple activity – retaining the segment managers and allocating resource (Assumption 2) – and (ii) the principal-agent relationship between the CEO and the segment managers that translates the segment managers' private benefits into the CEO's consideration.

After examining the total resource available to the firm, we will analyze the relationship between FRA and the CEO's ownership of the firm. Denote the equilibrium value of the multi-segment firm as W^{MX}. We know that

$$W^{MX} = \Phi\left[k_1^{MX}(\phi), f\left[\phi, k_1^{MX}(\phi)\right]\right] + b\left[k_1^{MX}(\phi)\right] + b\left\{f\left[\phi, k_1^{MX}(\phi)\right]\right\}.$$

The next proposition shows that social welfare, which equals the multi-segment firm's value, is increasing in the CEO's ownership.

Proposition 10 *(Inefficiency Increases in Separation of Ownership and Control)*

$$\frac{dW^{MX}}{d\phi} > 0.$$

Proposition 10 highlights one of the sources of inefficiency – separation of ownership and control. In general, this is not a surprising result, but we include it to show that our model is consistent with the empirical results of Scharfstein [26].

After understanding the role played by separation of ownership and control in FRA, we will turn to the next important source of the problem – diversification.

Diversification Destroys Value Since it is difficult to examine the welfare impact of diversification using the general functional form assumed above for V_i and

b, we will consider a special case of the model. Assume everything remains the same as before except that

$$V_1(k_1) = -ak_1^2 + bk_1 + \alpha k_1,\ V_2(k_2) = -ak_2^2 + bk_2 - \alpha k_2 \text{ and} \tag{17}$$

$$b(k_i) = -ck_i^2 + dk_i, \tag{18}$$

$$\text{where } \frac{b+\alpha}{a} < \frac{d}{c} \text{ and } \alpha > 0.$$

The parameter α serves as a measure of the degree of diversification for the multi-segment firm.

Denote the optimal value of the multi-segment firm as $\overline{W}$, and as a result $\overline{W} = \Phi(k_1^*, k_2^*) + b(k_1^*) + b(k_2^*)$. We believe what "diversification destroys values" means is that the *welfare loss,* $\overline{W} - W^{MX}$, is increasing in the degree of diversification, α in our context. This is shown in the next proposition:

Proposition 11 *(Diversification Destroys Value) If $V_i(k_i)$ and $b(k_i)$ satisfy equation (17) and (18), then the welfare loss of resource allocation is an increasing function of the degree of diversification of the firm, i.e.*

$$\frac{d\overline{W} - W^{MX}}{d\alpha} > 0,$$

1.2.3 Endogenous Management Ownership

In the following part of the analysis we will examine the effect of allowing stock-based compensation. It turns out that all the results developed in the previous section will hold qualitatively the same as long as the CEO does not end up capturing the ownership of the whole firm.

Optimal Resource Allocations in the Stand-alone Firm Suppose everything is the same as in section 2.2.1 except that the owner has one additional decision variable – the ownership of the CEO, ϕ_{C_i}. As a result, the two-tier principal-agent problem for each separately-owned firm i becomes finding $(S_{C_i}, k_i, \phi_{C_i})$ such that

$$(S_{C_i}, k_i, \phi_{C_i}) = \arg \max_{(\widetilde{S_{C_i}}, \widetilde{k_i}, \widetilde{\phi_{C_i}})} -\widetilde{S_{C_i}} + (1 - \widetilde{\phi_{C_i}})\Phi_i(\widetilde{k_i})$$

subject to

$$(\widetilde{\phi_{C_i}} - \phi_{M_i})\Phi_i(\widetilde{k_i}) + \widetilde{S_{C_i}} - S_i \geq \overline{u}_{C_i} \tag{19}$$

$$(S_i, \phi_{M_i}) = \arg \max_{(\widetilde{S_i}, \widetilde{\phi_{M_i}})} (\widetilde{\phi_{C_i}} - \widetilde{\phi_{M_i}})\Phi_i(\widetilde{k_i}) + \widetilde{S_{C_i}} - \widetilde{S_i}$$

subject to

$$\widetilde{S_i} \leq \widetilde{S_{C_i}}$$

$$\widetilde{S_i} + \widetilde{\phi_{M_i}}\Phi_i(\widetilde{k_i}) + b(\widetilde{k_i}) \geq \overline{u}_{M_i},$$

with all decision variables nonnegative. Define variables with superscript "SN" as their respective solutions in the stand-alone firm when the CEO's ownership is endogenous. Then the solution to this case is

$$(S_{C_1}^{SN}, S_{C_2}^{SN}, \phi_{C_1}^{SN}, \phi_{C_2}^{SN}, S_1^{SN}, S_2^{SN}, \phi_{M_1}^{SN}, \phi_{M_2}^{SN}, k_1^{SN}, k_2^{SN}),$$

where

$$k_i^{SN} = k_i^*,\ S_i^{SN} + \phi_{M_i}^{SN}\Phi_i(k_i^*) + b(k_i^*) = \overline{u}_{M_i}, \text{ and } (\phi_{C_i}^{SN} - \phi_{M_i}^{SN})\Phi_i(k_i^*) + S_{C_i}^{SN} - S_i^{SN} \geq \overline{u}_{C_i}, \tag{20}$$

for $i \in \{1,2\}$. Since the resource allocations are still the first best, allowing profit sharing does not matter for the stand-alone firm. In the next sub-section, we will turn to the case of the multi-segment firm.

Optimal Resource Allocations in the Multi-segment Firm Suppose everything is the same as in section 2.2.2 except that the principals, O and C, now have one additional decision variable – the ownership of the agents , ϕ_C for C and ϕ_{M_i} for M_i. As a result, the two-tier principal-agent problem for the multi-segment firm becomes finding (S, K, ϕ_C) such that

$$(S, K, \phi_C) = \arg \max_{(\widetilde{S}, \widetilde{K}, \widetilde{\phi_C})} -\widetilde{S} + (1 - \widetilde{\phi_C})\Phi(k_1, k_2) \tag{21}$$

subject to

$$\widetilde{\phi_C} \leq 1, \tag{22}$$

$$(\widetilde{\phi_C} - \phi_{M_1} - \phi_{M_2})\Phi(k_1, k_2) + \widetilde{S} - S_1 - S_2 \geq \overline{u}_C \tag{23}$$

$$(S_1, \phi_{M_1}, k_1, S_2, \phi_{M_2}, k_2) \tag{24}$$

$$= \arg \max_{(\widetilde{S_1}, \widetilde{\phi_{M_1}}, \widetilde{k_1}, \widetilde{S_2}, \widetilde{\phi_{M_2}}, \widetilde{k_2})} (\widetilde{\phi_C} - \widetilde{\phi_{M_1}} - \widetilde{\phi_{M_2}})\Phi(\widetilde{k_1}, \widetilde{k_2}) + \widetilde{S} - \widetilde{S_1} - \widetilde{S_2} \tag{25}$$

subject to

$$\widetilde{S_1} + \widetilde{S_2} \leq \widetilde{S},\ \widetilde{\phi_{M_1}} + \widetilde{\phi_{M_2}} \leq \widetilde{\phi_C},\ \widetilde{k_1} + \widetilde{k_2} \leq \widetilde{K}, \text{ and}$$

$$\widetilde{S_i} + \widetilde{\phi_{M_i}}\Phi(\widetilde{k_1}, \widetilde{k_2}) + b(\widetilde{k_i}) \geq \overline{u}_{M_i} \text{ for } i \in \{1,2\}, \tag{26}$$

with all decision variables nonnegative. Define variables with superscript "MN" as their respective solutions in the multi-segment firm when the CEO's ownership is endogenous. The following is the solution of this case.

Proposition 12 *If **Condition 1**: the CEO's participation constraint (equation (23)) is not binding and **Condition 2**: $[V_2'(k_2^{MN})-1]V_1'(k_1^{MN})-\Phi b''(k_2^{MN})<0$ hold, then $0<\phi_C^{MN}<1$ and the solution satisfies that*

$$S^{MN}=S_1^{MN}=S_2^{MN}=0, \tag{27}$$

$$\phi_{M_i}^{MN}=\frac{\overline{u}_{M_i}-b(k_i^{MN})}{\Phi(k_1^{MN},k_2^{MN})}>0 \text{ for } i\in\{1,2\}, \text{ and} \tag{28}$$

$$\phi_C^{MN}=\frac{[V_2'(k_2^{MN})-1]V_1'(k_1^{MN})-\Phi b''(k_2^{MN})}{[V_2'(k_2^{MN})-1]V_1'(k_1^{MN})+\Phi V_2''(k_2^{MN})}, \tag{29}$$

where $k_2^{MN}:=g\left(\phi_C^{MN},k_1^{MN}\right)$ is determined by

$$\phi_C^{MN}\left\{V_1'(k_1)-V_2'\left[g\left(\phi_C^{MN},k_1\right)\right]\right\}+b'(k_1)-b'\left[g\left(\phi_C^{MN},k_1\right)\right]=0 \tag{30}$$

and k_1^{MN} is determined by

$$V_1'\left(k_1^{MN}\right)-1+\left\{V_2'\left[g\left(\phi_C^{MN},k_1^{MN}\right)\right]-1\right\}\frac{\partial g\left(\phi_C^{MN},k_1^{MN}\right)}{\partial k_1}=0. \tag{31}$$

Otherwise, $\phi_C^{MN}=1$ and hence the resource allocations for both segments will be first best.

Since the main interest of this paper is resource misallocation caused by separation of ownership and control, we will focus our following discussion on the cases when $0<\phi_C^{MN}<1$, i.e. when Condition 1 and 2 hold.

The implication of equations (27) and (28) is that rewarding the CEO with shares is always preferred to cash. The reason is while both paying cash and paying shares help in retaining the CEO, the latter "currency" has an additional benefit – to reduce the separation of ownership and control. We summarize this implication with the next corollary.

Corollary 13 *When the owner can choose to reward the CEO with the firm's shares, paying cash will be replaced by paying shares.*

Note that in section 2.2 where managerial ownership is exogenous, if in equilibrium $S_1^{MX} = S_2^{MX} = 0$ the resource allocations will be first-best. The reason is because cash was the only currency that the segment managers can used to "trade" for an allocation different from the first best and the segment managers do not have any cash to do so when $S_1^{MX} = S_2^{MX} = 0$. However, in this section, $S_1^{MN} = S_2^{MN} = 0$ can be a equilibrium outcome when FRA arise because there are now two potential currencies – cash and shares – and the segment managers are given positive amount of shares (as in equation (28)) so that they still can trade for FRA even though they do not have any cash.

Some comparative-static results with respect to the equilibrium shares awarded to the CEO can be explored with equation (29) as shown in the next corollary.

Corollary 14 *The amount of shares the CEO receives, ϕ_C decreases in Φ and increases in $b''(k_2)$.*

The intuition behind Corollary 14 is the following: First, when Φ increases, given the same amount of shares, the value of the share rewarded to the CEO increases, and hence the owner can lower the amount of the award a bit without making the CEO's participation constraint bind. Second, when $b''(k_2)$ increases, it means it will require a bigger change in resource allocations to change a given amount of marginal utility on empire building. As a result, the misallocation problem becomes more severe because the CEO will need to flatten the resource allocations more in order to reduce the difference between the segment managers' marginal benefits from empire building (i.e. to achieve equation (30)). So it is conceivable that the CEO can appropriate more shares from the owner.

Another interesting result of Proposition 12 lies in Condition 1 – if we rule out the first-best case when the CEO capture the whole firm, the CEO is necessarily getting some positive "profit" (the part of utility that is over his reservation value) as his control rent independent of any possible competition in the CEO's labor market. This differs from the standard argument that a agent's participation constrains will bind when there is severe competition in the agent's labor market. There are some empirical results that are consistent with this result. For example, Bliss and Rosen [5] find that, in the banking industry, CEO compensation generally increases after mergers even if those mergers destroy value.

Furthermore, since managing the multi-segment firm gives the CEO more than what he can receive in the stand-alone firm, the next corollary follows directly:

Corollary 15 *When managerial ownership is endogenous (stock-based compensation is possible), the CEO will strictly prefer to manage the multi-segment firm.*

Finally and most importantly, equation (30) implies one of the central first-order conditions for FRA will still hold when the managerial ownership is endogenous as long as $\phi_C^{MN} < 1$. However, the pattern of "flattened" resource allocations become less clear-cut:

Proposition 16 (FRA and Diversification Discount) *When managerial ownership is endogenous, as long as the CEO does not capture the whole firm (* $\phi_C^{MN} < 1$*), then we have*

(a) the resource allocations are "flattened" in the following sense: Compared to the stand-alone firms, there exist either (I) underinvestment in the more profitable segment and overinvestment in the less profitable segment or (II) underinvestment in both segment. Specifically, the allocations satisfy one of the following three cases: (i) $k_2^- < k_2^* < k_2^{MN}$ *and* $k_1^{MN} < k_1^- < k_1^*$*, (ii)* $k_2^- < k_2^{MN} < k_2^*$ *and* $k_1^{MN} < k_1^- < k_1^*$*, or (iii)*$k_2^{MN} < k_2^- < k_2^*$ *and* $k_1^- < k_1^{MN} < k_1^*$*.*

(b) There exists diversification discount.

Regarding the total resource available to the firm, since the CEO is getting more than his reservation value, the owner's decision rule for the total resource available to the firm become equation (31), compared to equation (14) when the CEO's participation constraint binds. As a result, the condition for over- or underinvestment in the total resource available to the firm become the following:

Proposition 17 *There will be overinvestment in the total resource available to the firm if*

$$V_1'\left(\widehat{k}_1\right) - 1 + \left\{V_2'\left[g\left(\phi_C^{MN}, \widehat{k}_1\right)\right] - 1\right\} \frac{\partial g\left(\phi_C^{MN}, \widehat{k}_1\right)}{\partial k_1} > 0,$$

and there will be underinvestment in the total resource available to the firm if

$$V_1'\left(\widehat{k}_1\right) - 1 + \left\{V_2'\left[g\left(\phi_C^{MN}, \widehat{k}_1\right)\right] - 1\right\} \frac{\partial g\left(\phi_C^{MN}, \widehat{k}_1\right)}{\partial k_1} < 0.$$

Note that one part of the owner's marginal benefit of resource that appears in Proposition 9, $b'(k_1) + b'(k_2)\frac{\partial}{\partial k_1}k_2$, disappears in Proposition 17. This is because here the private benefit of the segment managers cannot get transmitted to the owner through the channel of compensation to the CEO, which is open only when the CEO's participation constraint is binding. Since $b'(k_1) + b'(k_2)\frac{\partial}{\partial k_1}k_2 > 0$, the owner's marginal revenue of resource is always greater when the owner cannot use stock-based compensation than when she can. The next result follows directly from this observation.

Corollary 18 *The owner will provide strictly less resource to the multi-segment firm when the owner can use stock-based compensation when she cannot.*

In the following sub-section, we will complete the analysis of the case of endogenous managerial ownership by reexamining the effect of diversification.

Diversification Destroys Value Denote the equilibrium value of the multi-segment firm as W^{MN}. Note that now $\overline{W} = \Phi\left(k_1^-, k_2^-\right)$ and

$$W^{MN} = \Phi\left[k_1^{MN}(\phi), g\left[\phi^{MN}, k_1^{MN}(\phi)\right]\right].$$

By using the quadratic functional form for $V_i(k_i)$ and $b(k_i)$ specified in equations (17) and (18), we can show that diversification still destroys value under endogenous manager ownership.

Proposition 19 *(Diversification Destroys Value) If $V_i(k_i)$ and $b(k_i)$ satisfies equations (*17*) and (*18*), then the welfare loss of FRA is an increasing function of the degree of diversification of the firm, i.e.*

$$\frac{d\overline{W} - W^{MN}}{d\alpha} > 0.$$

1.3 Discussion and Concluding Remarks

We can use the current static model to shed light on dynamic situations by interpreting the two different segments of a multi-segment firm producing in one period as one segment of a stand-alone firm producing in two different periods. With this interpretation, our results imply that the difficulty of the owner in controlling investment in one period from spilling over into another period, for example, through retained earnings, allows the CEO to flatten resource allocations between high- and low-productivity periods.

Two other related real-life phenomena deserve some discussion. First, even though many acquisitions destroy value, we often observe them taking place in the real world. Our explanation is that it is usually the CEO who dominates acquisition decisions. For instance, it is usually the CEO who has the best information on how much synergy can be generated by a certain merger. As long as, there is no strong consensus among the shareholders against the merger, the CEO will generally be able to get his way. According to Corollary 15, the CEO always prefers managing firms with more unrelated segments. So, they will proceed with acquisitions even though doing so might destroy the firm's value. Second, while conglomerates in general are the major victims of FRA, some conglomerates, for example General Electric, seem to perform very well even though the CEO has a small stake in the firm. As we understand, the long life of General Electric is primarily due to the excellent job of its legendary CEO, Jack Welch, in creating synergy by establishing a strong corporate culture in sharing valuable knowledge among segments[8]. When the synergy outweighs the cost of FRA, conglomerates could be efficient. While synergy is not modeled in the current model, it can generally be considered independently with the cost of FRA.

There seems to be a new kind of agency problem that has been identified – *moral*

[8] For detailed information on this, see "The house that Jack built", *Economist*, Sep 16th 1999.

hazard in hierarchy. There are several distinct characteristics of this kind of agency problem. First, it only exists in multiple-tier principal-agent relationships with multiple agents at the bottom tier. Second, allocating resource is one but not the only important activity that need to be carried out in the hierarchy. For instance, the CEO in our model is not only in charge of allocating resource but also retaining the segment managers. This creates the scope of inefficiently sacrificing one activity for the other. Third, hierarchical structure matters. For example, the stand-alone structure is preferred to the multi-segment structure in our setting. Finally, an agent whose objective is *misaligned* with the principal does not need to have direct access to the action that can be used to exploit the principal. While in most of the agency literature, problems occur because the objective of the principal and the agent misalign *and* the very same agent can use his action to pursuit personal interest at the expense of the principal. But in our model, the segment managers have the empire-building tendency but cannot alter their allocations. The CEO has the right to alter the allocations but does not care about empire building at all. In fact, before the CEO is assigned the job of retaining the segment managers, the objective of the owner and the CEO are the same – to maximize firm value, because they each own a portion of the firm. However, once the CEO is assigned as the principal of the segment managers, he starts to care about empire building because his goal becomes now to maximize the sub-social welfare instead of the total-social welfare.

To highlight the problem of FRA, we employed some rather extreme assumptions.

In practice, there are many observed bureaucratic restrictions which can alleviate the FRA problem, such as enhanced accounting and auditing practices which increase the *contractibility* of resource allocations and managers wages or limit the CEO's ability to divert personnel budget, or better separated authority in resource allocations and personnel retaining. However, as long as the contracting environment is not perfect, the problem of FRA will still exist.

This paper is one of the elementary steps in understanding the workings of the internal resource market. Obviously, a real-life internal resource market involves much more sophisticated features than we have presented in this paper. For example, there may well be multiple allocators in different layers of the hierarchy. Adding more allocators might enable us to analyze the cost and benefit of *decentralizing* the resource allocation decision. The next chapter presents an extension alone this line

2 Hierarchy Design with Flattened Resource Allocations

2.1 Introduction

One of the important agendas in organization theory is to understand why and how organization forms matter. While most organizations are *hierarchical* to some extent, they differ in many other respects. The primary focus of this paper is *hierarchy heights*, which is usually negatively correlated with another organizational variable – *average control span*. On this particular dimension, organizations can be categorized into the following two types: (1) *flat* hierarchies, the ones that have less layers and hence larger average control span, and (2) *tall* hierarchies, the ones that have more layers and hence smaller average control span. Flat hierarchies can be transformed into tall hierarchies through *divisionalization* – grouping elementary segments into smaller numbers of divisions and delegating some of the control of the divisions to division managers. In general, the heights of a hierarchy can represent the degree of delegation or decentralization and affects the efficiency of many activities such as information processing[9], intra-firm bargaining[10], capital budgeting[11], utilizing skilled worker[12], coordination and specialization[13], and protection of the source of organizational rent[14] in the organization.

The novel angle employed in this paper to examine the efficiency of different hi-

[9] There is a vast literature on this. See for example Bolton and Dewatripont[6].
[10] See Stole and Zwiebel [30].
[11] See Harris and Raviv [16].
[12] See Beggs[1].
[13] See Hart and Moore [13].
[14] See Rajan and Zingales[25].

erarchies is through *resource allocation*, one of the most important functions in most organizations, especially *the firm*. From perspectives slightly different from hierarchy heights, organization theorists have long been concerned about how organization forms affect the efficiency of resource allocations, especially the comparison between the *multi-division* or *M-form* firm and the *unitary form* or *U-form* firm. Known as the seminal *M-form Hypothesis*, Williamson [33, p150] argues that:

> The organization and operation of the large enterprise along the line of the M-form favors goal pursuit and least-cost behavior more nearly associated with the neoclassical profit maximization hypothesis than does the U-form organizational alternative.

According to Williamson, the M-form structure is more efficient than the U-form structure because by dividing the firm into divisions and delegating each division's functional decisions (such as manufacturing, sales, finance, and engineering) to the division managers, the M-form structure "economizes on bounded rationality" and hence enjoys lower information, fine-tuning, and displacement costs than U-form structure does in resource allocation. In short, the M-form firm can operate as a "miniature capital market" and has a better ability in assigning resource to high-yeild uses.

One might notice that there is a certain degree of overlapped focus in discussing organization forms and hierarchy designs. In fact, the distinguishing feature of fall hierarchies, compared to flat hierarchies, is *decentralized decision making*, which also happens to be one of the important features of the M-form structure that distinguish it from the U-form structure. From this perspective, discussions on the hierarchy heights can shed substantial light on organization forms.

Admittedly the M-form firm has many advantages in economizing bounded rationality in organizations. For a recent example, Maskin, Qian and Xu [21] argue that one of the superiorities of the M-form structure is that the M-form structure facilitates *relative performance evaluation*. However, as information technology and the external resource market advance, the rationality of the firm improves and hence the relative advantage of the M-form firm should diminish. In fact, recently many firms "reengineer" or "downsize" by eliminating middle-layer managers in order to survive competition,[15] suggesting that it can be more efficient for top managers to take back some of the decision rights that were delegated to the middle-layer managers. Also, firms with a tall hierarchy are getting replaced by firms with a flat hierarchy.[16] These facts seem to suggest that there might be a increasingly important cost in the M-form structure that has been long overlooked.

One crucial assumption underline the M-form Hypothesis is that *the more internal a organization's controlling apparatus is, the more efficient the organization will be*. This assumption is also used by Williamson to argue the superiority of the firm over the market in allocating resource because the firm has a more internal control than the market has. However, this assumption has been challenged by the recent developed literature on misallocation in internal resource (especially capital) market where

15 See Hammer and Champy[11].
16 See for example Buble[8].

many empirical evidences are found to show that the firm can actually underperform the market in resource allocation. For example Scharfstein [26] identify a peculiar inefficient pattern in the way multi-segment firms allocate resource inside the firm – low-profit segments tend to be overinvested and high-profit segments tend to be underinvested, that is, resource allocations are *flattened*. This finding cases serious doubt on Williamson's view about the firm. Moreover, it is also found that the seriousness of this flattened resource allocations (henceforth FRA) problem is positively correlated with the *diversity* of the firm and negatively correlated with the *managerial ownership*.[17] Again, all these evidences point to a cost in the firm that is rarely understood. Recently, the theoretical rationale for why FRA happen in the firm but not in the market has been explored by the previous chapter and Scharfstein and Stein [27]. In the current paper, we extend the previous chapter and reexamine the M-form hypothesis from the perspective of FRA.

The followings are our two main research questions: First, what is the potential pattern of FRA *when divisionalization is possible*? Second, and most importantly, how does the heights of a hierarchy affect its efficiency in resource allocation?

The answer to the first question the firm only has *two* business segments is provided by the previous chapter and Scharfstein and Stein [27]: it is simply that the high-profit segment gets underinvested and the low-profit segment gets overinvested. There is no important role for divisionalization in a two-segment firm because with or without

[17] The diversity result is found by Rajan, Servaes and Zingales [24], and the effect of managerial ownership is found by Scharfstein [26].

divisionalization it is still one person being in charge of the task of resource allocation. As a result there is always only one layer of resource allocations flattening. The only difference is that resource allocations are flattened by the CEO if the firm is not divisionalized and by the division manager if the firm is divisionalized.

Obviously, most multi-segment firms have more than two business segments in practice[18]. The pattern of FRA become subtler when the firm has many (more than two) business segments and divisionalization become an important issue because it will create new layers of allocations flattening under the existing one. The allocations flattening effect of the division manager necessarily affects the allocation decision of the CEO and hence the undivisionalized segments or segments belong to other divisions.

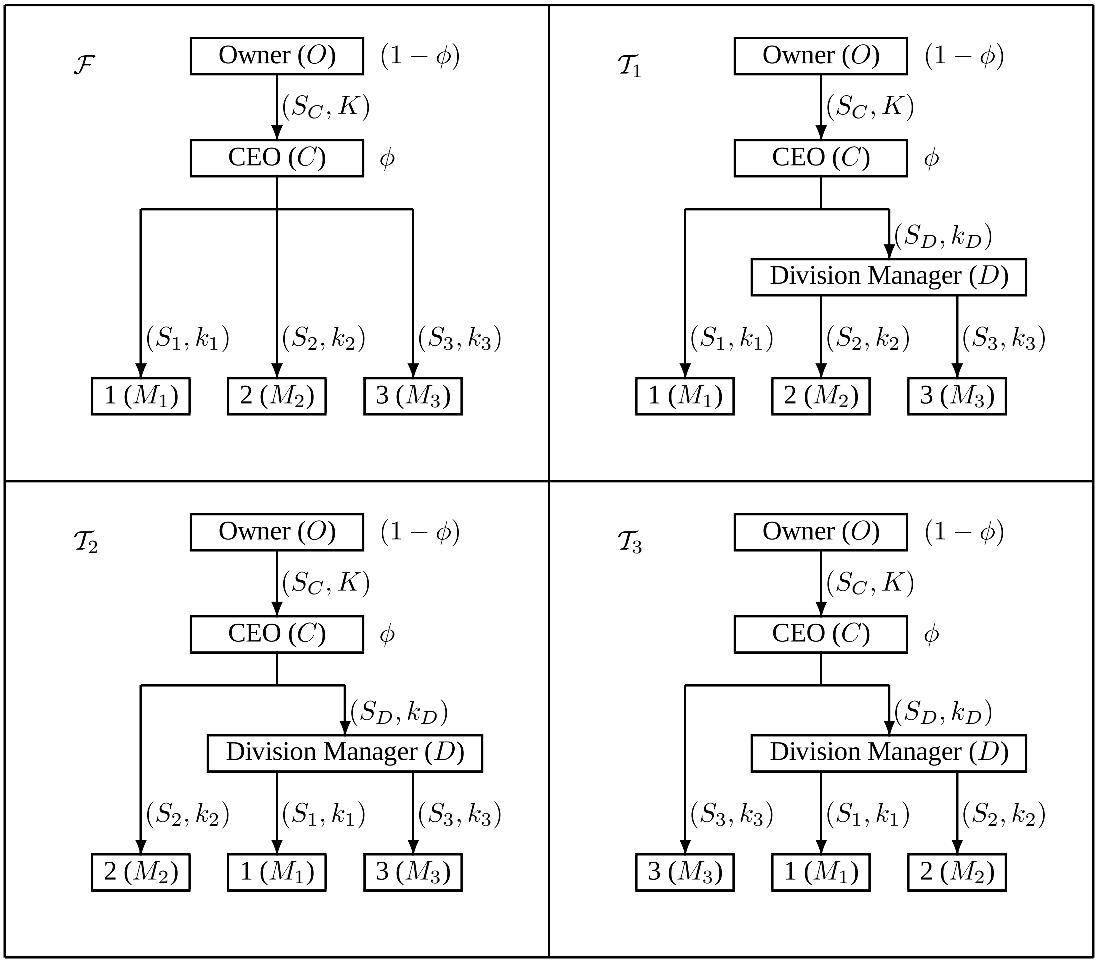
The answer to the second question is not trivial, either. While increasing the layers of a hierarchy would seems to entail additional *losses of control*,[19] it does not necessary do so.[20] In our context, adding layers to a hierarchy means replacing a "big" principal-agent problem with several "small" ones. Therefore, it is difficult to draw any conclusion with out working out a specific model.

We consider a firm with three different business segments (see Figure 3) that can be organized as a flat hierarchy, consisting one CEO directly controlling all three segments, or three different tall hierarchies, consisting on division manager directly

[18] In fact, some of them might even have hundreds or thousands of business segment under the name of one firm. According to *the Economist* (Sep. 1st-7th 2001), Toshiba, Fujitsu, and Hitachi respectively has 323, 517, and 1069 segments.
[19] See for example Williamson [35].
[20] See Mirrless [22]and Calvo and Wellisz[9].

controlling two segments and then a CEO controlling the division manager and the segment that does not belong to the division. Both the CEO and the division manager are agents of the firm's owner and are in charge of both allocating resource to and compensating their direct subordinates by paying them cash. In addition to the cash they received, the segment managers also have an *empire building tendency* and hence care about managing a segment allocated with as much resource as possible. Since a more profitable segment is supposed to receive more resource and the private benefit of empire building is a concave function of resource, a resource allocator can save cash by redirecting some resource from the segment manager who has lower marginal utility of resource (the manager the more profitable segment) to the segment manager who has higher marginal utility of resource (the manager of the less-profit segment), while keeping the segment managers receiving the same reservation values. Inefficient FRA is in the interest of the resource allocators because they do not bear the full cost of misallocation but enjoy the full benefit of cash payment saving. The previous chapter identifies the source of the problem of FRA as a negative externality caused by the resource allocator (the CEO) maximizing the welfare of a *sub-society* that excludes the owner without considering the cost to the whole society. In the current paper, an additional negative externality can added by divisionalization because the division manager will be maximizing the welfare of a even smaller sub-society that exclude, the owner, the CEO, and the manager of the segment that does not belong to the division. As a result, flat hierarchy outperforms any tall hierarchies in resources

Figure 3: Four Hierarchies



allocation because increasing the layer of a hierarchy necessarily aggravates the problem of FRA.

Generally speaking, the major differences between the U-form and the M-form structure is that most of the management decisions are made by the CEO in the U-form structure, whereas in the M-form structure some decisions (functional decisions) are delegated to lower-level division managers. In our setting, the tall hierarchy can be associated with the M-form structure since some of the decisions are delegated to lower-level division managers in the tall hierarchy, and the flat hierarchy can represents the U-form structure since it is only the CEO is making all the decisions in the flat hierarchy. With this linkage, our results suggest that *the U-form structure is more efficient than the M-form structure in resource allocation*, as opposed to Williamson's M-form hypothesis.

In fact, Williamson reckons that there are factors other than divisionalization and delegation that are also important for the M-form structure to realize its performance potential. However, what is surprising in our result is that when FRA arise, the M-form structure can not even work as efficiently as, let alone more efficiently than, the U-form structure. By concentrating on FRA, we highlight the possibility that even the firm might be able to develop an internal resource allocation capacity that can assign resources to high yield uses, it might not has the right incentives to do so. This is an inherent defect of the M-form structure that needs to be considered more seriously than before.

One might argue that, in the original text of Williamson, resource allocation was considered am example of the *strategic decisions* that need to be preserved for the CEO in both the U-form and M-form structure. So, in a sense, whether resource allocation is delegated can not seem to serve as distinction between the U-form and M-form structure. However, we believe this view is taking the text too literally. After all, it is hard to imagine the CEO of any M-form firm, especially big conglomerates that have hundreds of segments, is actually fine-tuning resource allocations to every elementary segment of the firm. What it really meant by strategic decisions are "big" decisions such as "allocation of resources among the competing operating *divisions*"[21] but not among elementary *segments* inside each division. Our result will be relevant as long the M-form structure facilitate a greater degree of delegation in resource allocation than the U-form structure does. While we admit that the tall and flat hierarchies in this model do not fit Williamson's original definition of U-form and the M-form structure word by word, the essential difference in the degree of delegation between the two organization forms is captured.

Most of the existing theoretical literature on FRA follows the *rent-seeking approach*, broadly defined as models that assume there exists some socially wasteful technology that segment managers can use to seek personal rent. For example, Rajan, Servaes and Zingales [24] assume that the managers of two segments can use the resource provided by the CEO to make wasteful "defensive investment" instead of efficient investment to protect their surplus from being poached by other segments. In another model, Scharfstein and Stein [27] assume that the managers of two segments, in stead of spending all of their effort in productive activities in current period, can spend some of their effort in unproductive rent-seeking activities (such as "resume polishing") which will improve their next period outside options. While these

[21] Quotation from Williamson [33], page 137.

model did provide useful insights to the problem, they share two major defects. First, and most importantly, their results are very sensitive to the detailed design of the rent-seeking game used in the models, implying a slight change of the model can overturn the results[22]. Second, the total resource available to the whole firm is exogenous. As a result, these models can not rule out the possibility that *all* segments are underinvested or overinvested, cases that are anomalies to FRA and can happen if the ultimate resource provider decide to underinvest or overinvest substantially in the firm. The previous chapter fixes these two problems and prove the results of FRA in a simpler model with two segments that does not involve any rent seeking behavior and endogenizes the total resource available to the firm.

In another rent-seeking model discussing hierarchy design, Inderst, Müller, and Wärneryd [18] argue that firms with more layers of hierarchy may experience lower influence activities. Combined with the results of the rent-seeking approach on FRA, this would seem to suggest that the FRA problem will be *less* severe in firms with *more* levels of hierarchy. This conjecture strengthens the motivation of the current paper since it contradict with our result and hence highlight the potential difference between our approach and the rent-seeking approach. However, given the problematic linkage between rent-seeking behavior and FRA, this conjecture is unlikely to be relevant.

As mentioned above, we obtain our result by extending the model of The previous

[22] For example, in Scharfstein and Stein [27] if one add the assumption that the segment managers's outside options are positively related with the segment's productivity, completely opposite results obtain.

chapter. An analogous research agenda is also followed in the related capital budgeting literature. Harris and Raviv [16] expended their earlier paper (Harris and Raviv [15]) that rationalizes the capital budgeting procedure and argue that delegation of resource allocation is more likely to be efficient when decentralized information is more costly to elicit or the value of the information is lower.[23] This result reconfirms the role of hierarchies in economizing information process cost and complement our result when information is decentralized. However, this capital budgeting literature can not address the problem of FRA because agency problems and diversity of the firm are the two major sources of FRA but do not play any role in the capital budgeting literature.

The rest of the paper is organized as follows: Section 2 setups the model. Section 3 analyzes the model and presents the results. Section 4 concludes the paper.

2.2 Model Setup

2.2.1 Assumptions and Definitions:

There are three business segments in a firm, indexed by i, $i \in \{1, 2, 3\}$. Denote the revenue function of segment i as $V_i(k_i)$, where k_i is the segment i's resource input. Each V_i satisfies the following assumption:

Assumption 1: $V_i : \mathbf{R} \to \mathbf{R}$, $i \in \{1, 2, 3\}$, *is a continuously differentiable strictly concave function which satisfies* $V_i(0) = 0$, $-\infty < V_i''(k) < 0$, and

$$V_3'(k) < V_2'(k) < V_1'(k). \tag{32}$$

[23] Recently Bernardo, Cai and Luo [4] extended Harris and Raviv's [16] model by incorporating moral hazard.

Inequality (32) reflects the diversity among the three segments, which is modeled by the differences in their marginal revenue of resource. By this construction, segment 1 is the most profitable segment, segment 3 is the least profitable segment, and segment 2 is the medium segment. The total profit of the firm is $V_1(k_1) + V_2(k_2) + V_3(k_3) - k_1 - k_2 - k_3$ and denoted as $\Phi(k_1, k_2, k_3)$.

There are four kinds of professionals in the firm: *the owner* (henceforth O), *the CEO* (henceforth C), *the division manager* (henceforth D), and *the segment managers* (henceforth M_i, $i \in \{1, 2, 3\}$). While the O, C, and M_i's are assumed to be indispensable to the firm, the existence of D need to be endogenously determined, depending on whether there is divisionalization. When there is no divisionalization and hence D is not employed, the firm consist of a two-tier hierarchy (as shown by $\mathcal{F}$ of Figure 3.), with O at the top as C's principal, C at the middle as M_i's principal and M_i's at the bottom of the hierarchy as simply agents. When there is divisionalization and hence D is hired, the firm consists of a three-tier hierarchy (as shown by $\mathcal{T}_1$, $\mathcal{T}_2$, or $\mathcal{T}_3$ of Figure 3.), with O at the top as C's principal, C at the second level as D's principal, D's at third level as the principal of those M_i's that belong to the division, and M_i's at the bottom level of the hierarchy as simply agents.

Except M_i's, all other professionals are potential shareholders of the firm. Denote total managerial ownership, the sum of shares owned by C and D, as $\phi < 1$. Consequently, the fraction of shares owned by O should be $1 - \phi$. For simplicity, we assume that M_i's possess no ownership of the firm. Relaxing this assumption will not change the result because, as it turns out, it is the total managerial ownership (ϕ) and its distribution between the decision makers, C and D, that matters. Denote also the fraction of managerial ownership owned by D as ρ. As a result, the ownership of C and D are $(1-\rho)\,\phi$ and $\rho\phi$, respectively. In this paper, ρ and ϕ are exogenous. Arguably, managerial ownerships can be decision variables of O. But, in practice, there are many reasons outside our model why the owner of a firm is refrained from freely rewarding the management with shares. One example is that the owner might want to maintain her status as a majority shareholder. Also, endoginizing ρ and ϕ is too difficult to analyze in the current framework and would not bring any qualitative changes to our results, as shown in the previous chapter with a less complex model.

While O, C, D, and M_i's all care about their personal wealth – the sum of the value of cash and shares they own, M_i's also enjoy a *private benefit*, $b(k_i)$ from managing a segment with resource k_i [24]. This private benefit reflects the *empire building tendency* of the segment managers and satisfies the following assumption:

Assumption 2: $b:\mathbf{R}\rightarrow\mathbf{R}$, *is a continuously differentiable function which satisfies*

$$\lim_{k_i\rightarrow 0+} b'(k_i)=\infty,\ b'(k_i)>0,\text{ and }-\infty<b''(k_i)<0.$$

O alone owns a unlimited endowment of cash and resource[25], whereas C, D, and M_i's have no endowment to start out with. In each layer of principal-agent relation-

[24] The possibility that O, C, or D might also enjoy private benefit from resource will not change the current result qualitatively and hence is ruled out for simplicity.

[25] One way to envision this is to imagine O as a group of dispersed shareholders who act cooperatively.

ship, the principal decides only how much cash and resource to give his/her *direct* agents *out of the cash and resource provided by the principal's principal*, if the principal is not O. If the principal is O, then she provides cash and resource out of her own endowment. By this setting, we implicitly invoke two important assumptions:

Assumption 3: *The amount of cash and resource a principal provides to each of his/her agent are not contractible to the principal's principal.*

Assumption 4: *A principal can divert any cash provided by his/her principal that are not given to his/her agents.*

"Resource" include anything that can be transformed into revenue, whereas "cash" is used to compensate employees to satisfy their participation constraints and is broadly defined to include *all means of compensating a employee*, such as salary, employee benefits, or cooperate perks. Assumption 3 can be rationalized by the well-known difficulty in attributing a firm's resource usage to each individual business segment in accurate accounting terms. Assumption 4 can be due to a budgeting system that facilitates flexibility. For example, the shareholder could allow a certain budget for employees' traveling. To the extent that a CEO can spend more on his own traveling, he might want to reduce the travel spending of his subordinates, given the assumption that everybody enjoys traveling.

Denote the cash that C got paid, D got paid, and M_i's got paid as, S_C, S_D, and S_i, respectively. To include all the possible hierarchy designs, we denote the segment that is directly managed by C (the "independent" segment) as segment l and the segments that are grouped into a division and managed by D (the divisionalized segments) as segments m and n with $n > m$ so that segment n (m) represents the divisionalized segment that is less (more) profitable. According our setup above, the utility functions for O, C, D, and M_i's are respectively

$$\begin{aligned} U^O &:= -S_C + (1-\phi)\Phi(k_l, k_m, k_n), \\ U^C &:= S_C - S_D - S_l + (1-\rho)\,\phi\Phi(k_l, k_m, k_n), \\ U^D &:= S_D - S_m - S_n + \rho\phi\Phi(k_l, k_m, k_n), \text{ and} \\ U^{M_i} &:= S_i + b(k_i). \end{aligned}$$

The first-best resource allocations are defined as follows:

Definition 6 *The first-best resource allocation for segment i is k_i^* such that*

$$k_i^* = \arg\max_{k_i}[V_i(k_i) + b(k_i) - k_i].$$

Assume also that for all i there exist $k_i < \infty$ such that $V_i'(k_i) + b'(k_i) < 1$[26]. According to the difference in the segments' marginal revenue, we have

$$k_1^* > k_2^* > k_3^*,$$

which reflects the idea that a more profitable segment usually deserves a higher resource allocation. The following four definition characterize FRA, which is complicated by the possibility of divisionalization.

Definition 7 *The resource allocations are flattened at the* ***division's*** *level if $k_n \geq k_n^*$ and $k_m \leq k_m^*$*

[26] This combined with the boundary condition in Assumption 2 guarantees that $k_i^* \in (0, \infty)$ for all i.

Definition 8 *The resource allocations are flattened at the* ***firm's*** *if* $k_3 \geq k_3^*$ *and* $k_1 \leq k_1^*$

Definition 9 *The resource allocations are* ***weakly*** *flattened if they are flattened at the division's level* ***or*** *at the firm's level.*

Definition 10 *The resource allocations are* ***strongly*** *flattened if they are flattened at the division's level* ***and*** *at the firm's level.*

To rule out non-instructive corner solutions where the CEO, division manager, or the segment managers receiving zero net cash, we also make the following assumption:

Assumption 5: *The reservation values of the CEO, the division manager and the segment managers are high enough (in particular strictly greater then zero) so that their participation can not be satisfied without being paid a positive net cash.*

With three segments, there are totally four possible hierarchies $\mathcal{F}$, $\mathcal{T}_1$, $\mathcal{T}_2$, and $\mathcal{T}_3$, as shown in Figure 3. Among them, $\mathcal{F}$ is the only *flat* hierarchy, representing U-form structure or organizations without divisionalization, whereas $\mathcal{T}_1$, $\mathcal{T}_2$, and $\mathcal{T}_3$ are *tall* hierarchies, representing M-form structure or organizations with divisionalization. The respective problems for flat and tall hierarchies are formulated in the following two subsections.

2.2.2 The Flat Hierarchy ($\mathcal{F}$)

Under $\mathcal{F}$, C directly manage three segment managers without the help of D. So in this case, ϕ is simply the ownership of C. Denote $\mathbf{S} := (S_1, S_2, S_3)$ and $\mathbf{k} := (k_1, k_2, k_3)$. To determine the resource allocations, we have to solve the following two-tier principal-agent problem:

$$\max_{(S_C,K)} -S_C + (1-\phi)\Phi(k_1,k_2,k_3)$$

subject to

$$S_C - \sum_{i=1}^{3} S_i + \phi\Phi(k_1,k_2,k_3) \geq \overline{u}_C \tag{33}$$

$$(\mathbf{S},\mathbf{k}) = \arg\max_{(\widetilde{\mathbf{S}},\widetilde{\mathbf{k}})} \left[S_C - \sum_{i=1}^{3} \widetilde{S}_i + \phi\Phi(\widetilde{k}_1,\widetilde{k}_2,\widetilde{k}_3) \right]$$

subject to

$$\sum_{i=1}^{3} \widetilde{S}_i \leq S_C$$

$$\sum_{i=1}^{3} \widetilde{k}_i \leq K \tag{34}$$

$$\widetilde{S}_i + b(\widetilde{k}_i) \geq \overline{u}_i \text{ for } i \in \{1,2,3\}, \tag{35}$$

with all decision variables non-negative and $\overline{u}_C$ and $\overline{u}_i$ respectively representing the reservation value of the CEO and the manager of segment i.

2.2.3 The Tall Hierarchy ($\mathcal{T}_l$)

For $\mathcal{T}_l$, the following three-tier principal-agent problem need to be solved:

$$\max_{(S_C,K)} -S_C + (1-\phi)\Phi(k_l,k_m,k_n)$$

subject to

$$S_C - S_D - S_l + (1-\rho)\,\phi\Phi(k_l,k_m,k_n) \geq \overline{u}_C \tag{36}$$

$$(S_D,S_l,k_D,k_l) = \arg\max_{\left(\widetilde{S_D},\widetilde{S}_l,\widetilde{k_D},\widetilde{k}_l\right)} S_C - \widetilde{S_D} - \widetilde{S}_l + (1-\rho)\,\phi\Phi(\widetilde{k}_l,k_m,k_n)$$

subject to

$$\widetilde{S_D} + \widetilde{S}_l \leq S_C \tag{37}$$

$$\widetilde{k_D} + \widetilde{k}_l \leq K \tag{38}$$

$$\widetilde{S_D} - S_m - S_n + \rho\phi\Phi(\widetilde{k}_l, k_m, k_n) \geq \overline{u}_D \tag{39}$$

$$(S_m, S_n, k_m, k_n) = \arg \max_{\left(\widetilde{S_m}, \widetilde{S_n}, \widetilde{k_m}, \widetilde{k_n}\right)} \widetilde{S_D} - \widetilde{S_m} - \widetilde{S_n} + \rho\phi\Phi(\widetilde{k}_l, \widetilde{k_m}, \widetilde{k}_n)$$

subject to

$$\widetilde{S_m} + \widetilde{S_n} \leq \widetilde{S_D} \tag{40}$$

$$\widetilde{k_m} + \widetilde{k}_n \leq \widetilde{k_D} \tag{41}$$

$$\widetilde{S_m} + b(\widetilde{k_m}) \geq \overline{u}_m \text{ and } \widetilde{S_n} + b(\widetilde{k}_n) \geq \overline{u}_n, \tag{42}$$

with all decision variables non-negative and $\overline{u}_C$, $\overline{u}_D$ and $\overline{u}_i$ respectively representing the reservation value of the CEO, the division manager and the manager of segment i.

2.3 The Analysis

The following two lemmas characterize the equilibrium for the resource allocations under flat and tall hierarchies.

Lemma 20 *The equilibrium resource allocations (k_1, k_2, k_3) for the **flat** hierarchy satisfy the first-order conditions for C's problem, which are*

$$\phi \left[V_2'(k_2) - V_1'(k_1)\right] + b'(k_2) - b'(k_1) = 0, \tag{43}$$

and

$$\phi \left[V_3'(k_3) - V_1'(k_1)\right] + b'(k_3) - b'(k_1) = 0, \tag{44}$$

and the first-order condition for O's problem, which is

$$V_1'(k_1) + b'(k_1) - 1 + [V_1'(k_2) + b'(k_2) - 1]\frac{\partial k_2}{\partial k_1} + [V_3'(k_3) + b'(k_3) - 1]\frac{\partial k_3}{\partial k_1} = 0. \tag{45}$$

Proof. All proofs of this chapter are in Appendix B. ■

Lemma 21 *The equilibrium resource allocations (k_1, k_2, k_3) for the **tall** hierarchy satisfy the first-order condition for D's problem, which is*

$$\rho\phi\left[V_m'(k_m) - V_n'(k_n)\right] + b'(k_m) - b'(k_n) = 0, \tag{46}$$

and the first-order condition for C's problem, which is

$$\begin{aligned} MB^C := \phi\left[V_m'(k_m)\frac{\partial k_m}{\partial k_n} + V_n'(k_n) - V_l'(k_l)\left(\frac{\partial k_m}{\partial k_n} + 1\right)\right] \\ + b'(k_m)\frac{\partial k_m}{\partial k_n} + b'(k_n) - b'(k_l)\left(\frac{\partial k_m}{\partial k_n} + 1\right) = 0, \end{aligned} \tag{47}$$

and the first-order condition for O's problem, which is

$$\begin{aligned} MB^O := V_l'(k_l) + b'(k_l) - 1 + [V_m'(k_m) + b'(k_m) - 1]\frac{\partial k_m}{\partial k_l} \\ + [V_n'(k_n) + b'(k_n) - 1]\frac{\partial k_n}{\partial k_l} = 0. \end{aligned} \tag{48}$$

Since the concavity of V_i's and b only guarantees the second-order conditions for equations (43), (44) and (46), we also need the following assumption to ensure the optimality of the solutions in Lemma 20 and 2:

Assumption 6:

$$\frac{\partial}{\partial k_n} MB^C < 0 \tag{49}$$

and

$$\frac{\partial}{\partial k_l} MB^O < 0. \tag{50}$$

One example where Assumption 6 holds is when functionsV_i's and b are quadratic[27].

[27] When functionsV_i and b are quadratic, $\partial MB^C/\partial k_n < 0$ since $\partial^2 k_m/\partial k_n^2 = 0$ and $\partial MB^O/\partial k_l < 0$ since $\partial^2 k_m/\partial k_l^2 = \partial^2 k_n/\partial k_l^2 = 0$ (can be obtained from equation (91)).

Proposition 22 *(a) The resource allocated to the three segments are positively related to each other, i.e.*

$$\frac{\partial k_m}{\partial k_n} > 0, \frac{\partial k_n}{\partial k_l} > 0 \text{ and hence } \frac{\partial k_m}{\partial k_l} > 0$$

(b) As D's portion of ownership ($\phi\rho$) increases, the resource allocated to the two divisionalized segments will move apart from each other, i.e.

$$\frac{\partial k_m}{\partial \rho} > 0 \text{ and hence } \frac{\partial k_n}{\partial \rho} < 0; \frac{\partial k_m}{\partial \phi} > 0 \text{ and hence } \frac{\partial k_n}{\partial \phi} < 0.$$

Part (b) of Proposition 22 confirms the result of Scharfstein [26] and the intuition that reduced separation of ownership from control results in more efficient allocations[28] since, generally speaking, less flattened allocations are more efficient.

One important problem in the internal resource market is that segments can not get their optimal resource allocations based on their *own* profit-maximizing concerns. The resource allocated to one segment necessarily depends on attributes, such as cash flow (Shin and Stulz [28]), of the *other* segments in the firm. This externality is previously modeled in the previous chapter. We extended it in the following result by incorporating divisionalization:

Proposition 23 *<Socialism in the Internal Resource Market> Any change in the total resource available to the firm (division) will cause the resource allocation of each segment in the firm (division) to change in the same direction, i.e.*

$$0 < \frac{\partial k_l}{\partial K} < 1, 0 < \frac{\partial k_D}{\partial K} < 1, 0 < \frac{\partial k_m}{\partial k_D} < 1, \text{ and } 0 < \frac{\partial k_n}{\partial k_D} < 1.$$

We characterize a general pattern of FRA for all the hierarchies in the next result.

Proposition 24 *The resource allocations are strongly flattened in $\mathcal{F}$ and strongly flattened in $\mathcal{T}_l$. In particular, segment 1 (the most profitable segment) is always underinvested, i.e. $k_1 < k_1^*$, except in $\mathcal{T}_1$; segment 3 (the least profitable segment) is always overinvested, i.e. $k_3 > k_3^*$, except in $\mathcal{T}_3$.*

[28] This is statement is formally proved in Proposition 10.

The reason why there could be cases that resource allocations are only weakly flattened but not strongly flattened is because the resource flattening of the division manager complicates the resource allocations flattening of the CEO. As previously shown in the previous chapter, there could be overinvestment or underinvestment in the total resource available to the firm when the CEO is the only one flattening the resource allocations. In the current model, the division manager also has the ability to flatten resource allocations inside the division and hence there can also be overinvestment or underinvestment in the total resource available to the *division*. As a result, it could be optimal to overinvest (underinvest) in segment 1 (3) if the underinvestment (overinvestment) in the total resource available to the division is so severe that the benefit of increasing (decreasing) the total resource available to the division out weights the cost of overinvesting (underinvesting) in segment 1 (3), since the resource allocations to the division and the independent segment, k_D and k_I respectively, should always move in the same direction as shown in Proposition 22.

The next result characterize FRA for each tall hierarchy.

Proposition 25 *(a) Under $\mathcal{T}_2$, the resource allocations are strongly flattened.*
(b) Under $\mathcal{T}_1$ ($\mathcal{T}_3$), the resource allocations are flattened at the division's level if they are flattened at the firm's level and ϕ is big enough.

Denote $W^{\mathcal{F}}(\phi)$ as the equilibrium value of the firm under $\mathcal{F}$ with managerial ownership ϕ and $W^{\mathcal{T}_l}(\phi, \rho)$ as the equilibrium value of the firm under hierarchy $\mathcal{T}_l$ with managerial ownership ϕ and division manager owning ρ of the managerial ownership. In the next result, we establish a way to compare the efficiency of $\mathcal{F}$ and $\mathcal{T}_l$ by showing that the allocations under $\mathcal{F}$ is a special case of the allocations under $\mathcal{T}_l$.

Proposition 26 *When D owns 100% of the managerial ownership ($\rho = 1$), all hierarchies perform equally efficiently, i.e.*

$$W^{\mathcal{F}}(\phi) = W^{\mathcal{T}_l}(\phi, 1) \text{ for } l \in \{1, 2, 3\}.$$

Proposition 26 establishes a important observation that allows us to compare $\mathcal{F}$ and $\mathcal{T}_l$ without calculating out their respective values. In the following proposition we establish our key results:

Proposition 27 *(a). The equilibrium firm value increases in the managerial ownership, i.e. $\partial W^{\mathcal{T}_l}(\phi, \rho)/\partial \phi > 0$.*

(b). The equilibrium firm value increases in the D's portion of managerial ownership, i.e. $\partial W^{\mathcal{T}_l}(\phi, \rho)/\partial \rho > 0$ and, as a result, $\mathcal{F}$ ***is more efficient than*** *$\mathcal{T}_l$.*

While both parts (a) and (b) of Proposition 27 simply reconfirm the intuition that increased managerial ownership improve efficiency, part (b), combined with Proposition 26, also implies that *divisionalization destroys value*.

2.4 Conclusion

This paper extended Chou's [**?**] model of FRA to examine the efficiency of different hierarchy designs. It is shown that flat hierarchies are more efficient in resource allocation than tall hierarchies because increasing the layer of hierarchies will cause a additional negative externality in resource allocation and hence aggravate the problem of FRA. This result suggests that the U-form structure can out-perform the M-form structure – as opposite to Williamson's view that the M-form structure prevails because it is more capable in assigning resource to high-yield uses than the U-form structure is.

3 The Boundaries of the Firms in the Absence of Property Rights

3.1 Introduction

A vast literature of property right approach to the theory of the firm has been established since the seminal papers of Grossman, Hart and Moore (henceforth GHM, including Grossman and Hart[10] and Hart and Moore[12]). One of the general theme of this literature is that, when contracts are incomplete, ownership of physical assets can matter because it can alter the marginal return to investment and hence change the investors incentive to invest. One obvious and important limitation of this approach is that it does not apply to human capital intensive firms, such as law, consulting, and high-tech firms, because human capital is simply inalienable. Therefore, as advocated by Zingales[36], a new theory of the firm is warranted to deal with the increasingly important human capital intensive firms. This paper presents a very simple model in this direction, using the same incomplete-contract methodology of GHM, and shows how classical questions such as what is a firm? and what determines the boundaries of the firm? can be answered when there is no physical asset.

What motivates this paper is the following observation of the real world firm: when people work inside the boundaries of one firm the values of their individual human capital are usually harder to access for the external labor markets then when people work as many separated firms. The external labor market can only estimate the value of each employee's human capital based on the market's belief about how much each employee contribute in different components of the firm's output. This suggests the boundaries of the firms can serve as information barriers that prevent inside information about individual employees from being clearly observed by outside employers. This observation also naturally leads to the following question: Is this unobservability of the external labor market a cost or a benefit? We find that both situation can be the case.

We should use the following example in economics research to help illustrating the idea. Suppose solving some economic problem requires a model and an empirical test that can be produced by two economists A (a theorist) and B (a econometrician), respectively. The total value of solving the problem equals the sum of the values of the model and the test which are both increasing functions of the investments of both economists. There are two ways A and B can work on the problem: to publish two single-author papers separately (one with the model and the other one with the test) or to publish one joint paper. The former represents the case when two people work as independent firms and will be refereed as 'separation.' The latter represents the case when two people work inside the boundaries of the firm and will be refereed as 'integration.'

After the research is done (published), A and B can capitalize their idea by consulting for the government on how to implement the solution. The implementation requires the human capital of both of the two economist and will not be accomplished by any single one of them. If A and B agree to consult, the problem is solved and a reward which equals to the sum of the values of the model and the test will be awarded jointly to A and B by the government. However, if they don't agree, they can independently use their respective human capital to teach in a university, which does not require any agreement between A and B. Assume that after they both made their investments A and B bargain over the total consulting reward in a 50-50 Nash bargaining game with their respective teaching rewards as reservation values.

How is GHM approach supposed to work and why can't it work in our example? As in all 50-50 Nash bargaining game, the total payoff of a economist is the sum of the following two parts: (i) his 'teaching payoff' (or 'no-trade payoff' as generally referred in the literature) and (ii) 50% of the difference of the total reward for two of them between consulting and teaching. This implies that any factor that affect the marginal return in teaching will affect A and B's marginal returns in consulting, even though teaching will never be a equilibrium action of A and B, because the bargaining structure entails teaching payoffs enter (positively) into each economist's objective function. The GHM intuition is that if ownership of physical assets, for example computers, is complementary in teaching to A and B's investment, then ownership can induce higher investment. This intuition does not apply to human capital intensive industries because ownership of physical asset should has very small influence on investment incentives. Instead, what might matter is the way A and B work with each other: separation or integration.

If A and B separate, the university can easily identified who did the model and who did the test and hence can attribute the value of the model and test to their respective producer. On the other hand, if A and B integrate, it is more difficult for the university to identified who did what and hence A and B will be paid with the estimated value of their human capitals in teaching based on the university's belief about each economist's contribution to the model and the test. For example if the university believe that B contribute 20% of the model and 70% of the test, then the value of B's human capital equals 20% of the model's value plus 70% of the test's value. To reflect the idea that A and B's investments are relation-specific investment in consulting, we assume that the university is only willing to pay each economists a portion say 90% of the estimated value of each economist's research[29].

Recall that the total payoff of a economist is the sum of his teaching payoff and 50% of the difference of total reward for two of them between consulting and teaching. Since the latter term will be the same no matter A and B separate or integrate, the difference of incentives between the two cases lies in the first term which equals 90% of the marginal return of A and B'srespective products if they separate or 90% of a mixture of the marginal returns of the model and test according to the university's belief about each economist's contribution in the two product if A and B integrate . For example, if they integrate and the university believe B contribute 20% of the model and 70% of the test, then B's marginal return in teaching will be 90 % of the sum of 20% of the model's marginal return and 70% of the test's marginal return.

[29] This could be because teaching is a less profitable use of the idea.

By assuming that marginal returns are decreasing and marginal costs of investments are constant, we can compare the equilibrium investment level under separation and integration by simply comparing each economist's marginal return in the two cases. In the example mentioned above, integration will provide better incentive for B to invest if (100%) of the marginal return of the test is lower than the sum of 20% of the model's marginal return plus 70% of the test's marginal return. Obviously, this can only be the case when the model's marginal return from B's investment is large enough, i.e. there is a substantial positive externality in B's investment to the value of the model. Given that there is always underinvestment due to the hold-up problem and hence whichever way of publishing gives better investment incentive will be more efficient, we can see that publishing a joint paper will be more efficient than publishing two separated papers when there exists substantial externality in A and B's investments.

This paper is still a work in progress at this moment with mostly a preliminary model in section 2. Literature review and conclusion will be added latter.

3.2 Model

A primary project requires the investment $a \geq 0$ and $b \geq 0$ of two persons A and B, respectively. The private marginal costs of investments are unity. Given a and b, the values of A and B's human capitals worth $A(a, b)$ and $B(a, b)$, respectively if the project is carried out, with $A_a > 0$ and $B_b > 0$. Notice that one person's investment will affect the value of the value of the other person's human capital, i.e.

there exists investment externality between A and B. For simplicity, we will focus on positive investment externality by assuming that $A_b > 0$ and $B_a > 0$. Furthermore, we assume that $A_{aa} < 0$, $A_{bb} < 0$, , $B_{aa} < 0$, and $B_{bb} < 0$.

The primary project is a joint production of A and B in the sense that it requires both A and B to agree to use their human capital in the project. If they do agree, the a return which equals $A(a,b) + B(a,b)$ will be paid jointly to A and B. If they do not agree, each of them can use his human capital independently in a secondary project. However, the values of A and B's human capital will be discounted in their respective secondary project because these secondary projects are less efficient uses of their human capitals. Denote the values of A and B's human capitals in the secondary projects as $\underline{A}(a,b)$ and $\underline{B}(a,b)$, respectively, and assume that $\underline{A} = \phi A$ and $\underline{B} = \phi B$ where $\phi \in (0,1)$. A and B know that it is jointly more profitable form them to agree on the primary project but need to decide how to split the joint return $A(a,b) + B(a,b)$. Specifically, we assume that *after* A and B both made their investments they bargain over $A(a,b) + B(a,b)$ in a 50-50 Nash bargaining game with their payoffs in secondary projects as reservation values.

There are two ways for A and B to work (make investment): to *separate* and work as two firms or to *integrate* and work as one firm. The payoffs for A and B in their secondary projects depend on whether they separate or integrate. Denote $\alpha \in [0,1]$ as the probability the clients of the secondary projects believe A is actually the owner of the human capital that is worth $\underline{A}$ and $\beta \in [0,1]$ as the probability the clients believe

B is actually the owner of the human capital that is worth $\underline{B}$. Assume the payoffs for A and B in their secondary projects equal the clients' estimations of the worth of A and B's human capital. As a result, A and B will receive $\alpha\underline{A} + (1-\beta)\,\underline{B}$ and $(1-\alpha)\,\underline{A} + \beta\underline{B}$ respectively in their secondary projects. As the central idea of the paper, we distinguish separation with integration by assuming that $\alpha = 1$ and $\beta = 1$ if A and B separate, whereas $\alpha < 1$ and $\beta < 1$ if A and B integrate. This assumption reflect the idea that when A and B separate their clients can clearly observe the value of their individual human capitals whereas when A and B integrate their individual identities are blurred and hence the clients can only make estimation about the value of A and B's human capital with potentially small but positive possibility that one of them is owning the other's human capital. As a result, the estimated value of one's human capital become a mixture of the value of human capitals of everyone in the firm. Finally, note that separation and integration are two scenario that we wish to compare. They are not a choice of any player in the model.

We will set our benchmark as the first best investment which is a pair (a^*, b^*) such that

$$(a^*, b^*) = \arg\max A + B - a - b$$

or

$$A_a\,(a^*, b^*) + B_a\,(a^*, b^*) = 1$$

and

$$A_b\,(a^*, b^*) + B_b\,(a^*, b^*) = 1.$$

3.2.1 Separation

When A and B are separated, 50-50 Nash bargaining implies that A will get

$$\underline{A}+\frac{1}{2}\left[A+B-\underline{A}-\underline{B}\right]-a$$

and B will get

$$\underline{B}+\frac{1}{2}\left[A+B-\underline{A}-\underline{B}\right]-b$$

In this case, equilibrium investments are a^S and b^S such that

$$\frac{1}{2}\left[A_a\left(a^S,b^S\right)+B_a\left(a^S,b^S\right)+\underline{A}_a\left(a^S,b^S\right)-\underline{B}_a\left(a^S,b^S\right)\right]=1$$
$$\frac{1}{2}\left[A_b\left(a^S,b^S\right)+B_b\left(a^S,b^S\right)-\underline{A}_b\left(a^S,b^S\right)+\underline{B}_b\left(a^S,b^S\right)\right]=1$$

The first observation is that there will be underinvestment problem due to the hold-up problem:

Proposition 28 *$a^S<a^*$ and $b^S<b^*$*

Proof. Since $\frac{1}{2}\left[A_{aa}+B_{aa}+\underline{A}_{aa}-\underline{B}_{aa}\right]=\frac{1}{2}\left[(1+\phi)A_{aa}+(1-\phi)B_{aa}\right]<0$, $\frac{1}{2}\left[A_a+B_a+\underline{A}_a-\underline{B}_a\right]$ is decreasing in a. Therefore, $a^S<a^*\Longleftarrow\underline{A}_a-\underline{B}_a<A_a+B_a\Longleftarrow\phi\left(A_a-B_a\right)<A_a+B_a\Longleftarrow\left(A_a-B_a\right)<A_a+B_a$. ■

3.2.2 Integration

When A and B are integrated, 50-50 Nash bargaining implies that A will get

$$\alpha\underline{A}+(1-\beta)\underline{B}+\frac{1}{2}\left[A+B-\underline{A}-\underline{B}\right]-a$$

and B will get

$$(1-\alpha)\underline{A}+\beta\underline{B}+\frac{1}{2}\left[A+B-\underline{A}-\underline{B}\right]-b$$

Equilibrium investments are a^I and b^I such that

$$\frac{1}{2}\left(A_a+B_a\right)+\left(\alpha-\frac{1}{2}\right)\underline{A}_a-\left(\beta-\frac{1}{2}\right)\underline{B}_a=1 \tag{51}$$

$$\frac{1}{2}\left(A_b+B_b\right)-\left(\beta-\frac{1}{2}\right)\underline{A}_b+\left(\alpha-\frac{1}{2}\right)\underline{B}_b=1 \tag{52}$$

The followings are investment level comparison between integration and first best and between integration and separation.

Proposition 29 *1) $a^I<a^*$ and $b^I<b^*$.*
2) $\underline{A}_a<\alpha\underline{A}_a+(1-\beta)\underline{B}_a\Leftrightarrow a^S<a^I$ and $\underline{B}_b<(1-\alpha)\underline{A}_b+\beta\underline{B}_b\Leftrightarrow b^S<b^I$.

Proof. It suffices to prove the part for A

1).Since

$$\begin{aligned}&\frac{1}{2}\left[A_{aa}+B_{aa}\right]+\left(\alpha-\frac{1}{2}\right)\phi A_{aa}-\left(\beta-\frac{1}{2}\right)\phi B_{aa}\\=&\underbrace{\left(\frac{1}{2}+\alpha\phi-\frac{1}{2}\phi\right)}_{(+)}A_{aa}+\underbrace{\left(\frac{1}{2}-\left(\beta-\frac{1}{2}\right)\phi\right)}_{(+)}B_{aa}<0,\end{aligned}$$

$\frac{1}{2}\left[A_a+B_a\right]+\left(\alpha-\frac{1}{2}\right)\underline{A}_a-\left(\beta-\frac{1}{2}\right)\underline{B}_a$ is decreasing in a. As a result,

$$\begin{aligned}&a^I<a^*\\\Leftarrow&\frac{1}{2}\left[A_a+B_a\right]+\left(\alpha-\frac{1}{2}\right)\underline{A}_a-\left(\beta-\frac{1}{2}\right)\underline{B}_a<A_a+B_a\\\Leftarrow&\underbrace{\left(\left(\alpha-\frac{1}{2}\right)\phi-\frac{1}{2}\right)}_{(-)}A_a<\underbrace{\left(\left(\beta-\frac{1}{2}\right)\phi+\frac{1}{2}\right)}_{(+)}B_a\end{aligned}$$

2). Since the marginal returns of a under separation and integration are both decreasing functions of a, the comparison of equilibrium investment can be obtain by comparing the marginal returns of separation and integration. ■

Many examples can be found in elementary economic textbooks that illustrate the concept that institutions likes merger exist to internalize externality. The general intuition is that inefficiency arises when two firms operate independently because each firm will not consider the externality of its action to the other firm and integration of the two firms achieves efficiency because the owner of the integrated firm will choose to maximize joint surplus and hence internalize the externality. It is important to note that the example above works only for physical asset intensive firms because then integration can change a two-person noncooperative game (two separated firms) into a one-person decision problem (one integrated firm.) When the two firms are human capital intensive, as the case that we are interested in this paper, the intuition is less trivial because under integration there are still two persons making decisions about their human capitals. The following result reconfirms the intuition:

Corollary 30 *Integration is efficient when there exists substantial investment externality.*

Proof. When A_b and B_a are big enough such that $\alpha \underline{A}_a + (1-\beta)\underline{B}_a = \alpha \underline{A}_a > \underline{A}_a$ and $(1-\alpha)\underline{A}_b + \beta \underline{B}_b = \beta \underline{B}_b > \underline{B}_b$, we have $a^I > a^S$ and $b^I > b^S$. ■

The two essential factors at work in the model are hold-up problem and investment externality. The former causes underinvestment in both separation and integration.

The latter, however, can serve as a remedy to the underinvestment problem that is better utilized by integration. The intuition behind Corollary 30 is that integration work best when the there is a strong remedy to work with

The following is a partial ordering between separation and integration which follows directly from Proposition 29:

Corollary 31 *If* $\underline{A}_a < \alpha \underline{A}_a + (1-\beta)\,\underline{B}_a$ *and* $\underline{B}_b < (1-\alpha)\,\underline{A}_b + \beta \underline{B}_b$*, then integration is efficient. If* $\underline{A}_a > \alpha \underline{A}_a + (1-\beta)\,\underline{B}_a$ *and* $\underline{B}_b > (1-\alpha)\,\underline{A}_b + \beta \underline{B}_b$*, then separation is efficient.*

In the following results, we can characterize the implementable set of investments and the condition under which the first best can be implemented. By solving the F.O.C.'s for α and β, we have

$$\alpha(a,b) = \frac{(\phi-1)}{2\phi} + \frac{A_b(a,b) - B_a(a,b)}{\phi\left(A_b(a,b)\,A_a(a,b) - B_b(a,b)\,B_a(a,b)\right)}$$

and

$$\beta(a,b) = \frac{(\phi+1)}{2\phi} + \frac{B_b(a,b) - A_a(a,b)}{\phi\left(A_b(a,b)\,A_a(a,b) - B_b(a,b)\,B_a(a,b)\right)}.$$

Proposition 32 *If* $\alpha(a,b) \in [0,1]$ *and* $\beta(a,b) \in [0,1]$*, then* a *and* b *are implementable in integration with* $\beta(a,b)$ *and* $\alpha(a,b)$*.*

Corollary 33 *If* $\alpha(a^*,b^*) \in [0,1]$ *and* $\beta(a^*,b^*) \in [0,1]$ *, then the first best can be implemented by integration.*

3.3 Discussion

It is important to note that the definition of the firm in this paper can be very different from the existing GHM point of view, in which a firm contains exactly one people:

two firms from the GHM point of view that always works together and hence does not have clearly defined individual image can be considered as one firm in our point of view.

4 Appendix A

Proof of Proposition 1. The CEO's problem, given the owner's decision of $(\widetilde{S}, \widetilde{K})$, is to find (S_1, S_2, k_1, k_2) such that

$$(S_1, S_2, k_1, k_2) = \arg \max_{(\widetilde{S_1}, \widetilde{S_2}, \widetilde{k_1}, \widetilde{k_2})} \phi\Phi(\widetilde{k_1}, \widetilde{k_2}) + \widetilde{S} - \widetilde{S_1} - \widetilde{S_2} \tag{53}$$

subject to

$$\widetilde{S_1} + \widetilde{S_2} \leq \widetilde{S},$$

$$\widetilde{k_1} + \widetilde{k_2} \leq \widetilde{K} \tag{54}$$

$$\widetilde{S_i} + b(\widetilde{k_i}) \geq \overline{u}_{Mi} \text{ for } i \in \{1, 2\}. \tag{55}$$

Since equation (55) will hold as equality in equilibrium, we have a

$$S_i = \overline{u}_{Mi} - b(k_i) \text{ for } i \in \{1, 2\}. \tag{56}$$

Also, since the CEO does not have incentive to leave any resource not invested in one of the segments, equation (54) will also holds as equality. By plugging equation (56) and $\widetilde{k_2} = \widetilde{K} - \widetilde{k_1}$into equation (53), the CEO's problem becomes to find (S_1, S_2, k_1) such that

$$(S_1, S_2, k_1) = \arg \max_{(\widetilde{S_1}, \widetilde{S_2}, \widetilde{k_1})} \phi\Phi(\widetilde{k_1}, \widetilde{K} - \widetilde{k_1}) + \widetilde{S} + b(\widetilde{k_1}) + b(\widetilde{K} - \widetilde{k_1}) - \mathfrak{C}_2$$

subject to

$$\widetilde{S_1} + \widetilde{S_2} \leq \widetilde{S},$$

$$\widetilde{k_1} \leq \widetilde{K}$$

where $C_2 = \overline{u}_{M_1} + \overline{u}_{M_2}$. The Lagrangian function of the CEO's problem is

$$\mathcal{L}(S_1, S_2, k_1, \lambda_1, \lambda_2)$$

$$= \phi\Phi(k_1, K - k_1) + S + b(k_1) + b(K - k_1) - \mathfrak{C}_2 + \lambda_1[S - S_1 - S_2] + \lambda_2[K - k_1].$$

Note that $\lambda_2 = 0$ since $\lim_{k_i \to 0+} b'(k_i) = \infty$. The Kuhn-Tucker Conditions are:

$$\mathcal{L}_{S_i} = -\lambda_1 \leq 0 \text{ if } S_i \geq 0 \text{ and } S_i\mathcal{L}_{S_i} = 0 \text{ for } i \in \{1, 2\}$$

$$\mathcal{L}_{k_1} = \frac{\partial}{\partial k_1}\{\phi[V_1(k_1) + V_2(K - k_1)] + b(k_1) + b(K - k_1)\}$$
$$= \phi[V_1'(k_1) - V_2'(K - k_1)] + b'(k_1) - b'(K - k_1) \leq 0 \text{ if } k_1 \geq 0 \text{ and } k_1\mathcal{L}_{k_1} = 0 \tag{57}$$
$$\mathcal{L}_{\lambda_1} = S - S_1 - S_2 \geq 0 \text{ if } \lambda_1 \geq 0 \text{ and } \lambda_1\mathcal{L}_{\lambda_1} = 0$$

Note that $k_i > 0$ since $\lim_{k_i \to 0+} b'(k_i) = \infty$. The complementary slackness condition of equation (57) implies that

$$\phi[V_1'(k_1) - V_2'(K - k_1)] + b'(k_1) - b'(K - k_1) = 0. \tag{58}$$

Therefore, given the owner's decision of (S, K), the solution to the CEO's problem is

$$(S_1, S_2, k_1, k_2) = (\overline{u}_{M_1} - b(k_1), \overline{u}_{M_2} - b(k_2), k_1, \widetilde{K} - k_1),$$

where k_1 satisfies equation (58).

Proof of Corollary 2. Since $S_i^{MX} = \overline{u}_{M_i} - b(k_i^{MX})$ and $S_i^{SX} = \overline{u}_{M_i} - b(k_i^{SX})$, the resource allocations are flattened if and only if

$$k_1^{MX} < k_1^{SX} \text{ and } k_2^{MX} > k_2^{SX}$$

$$\Leftrightarrow \overline{u}_{M_1} - b(k_1^{MX}) > \overline{u}_{M_1} - b(k_1^{SX}) \text{ and } \overline{u}_{M_2} - b(k_2^{MX}) < \overline{u}_{M_2} - b(k_2^{SX})$$

$$\Leftrightarrow S_1^{MX} > S_1^{SX} \text{ and } S_2^{MX} < S_2^{SX}.$$

Proof of Proposition 3. By applying *the implicit-function theorem* to equation (12), we can write k_2^{MX} as a function of ϕ and k_1^{MX}, i.e. $k_2^{MX} := f(\phi, k_1^{MX})$. Also, the following partial derivatives are well-defined and can be obtained by *the implicit-function rule*:

$$\begin{aligned}
\frac{\partial k_1^{MX}}{\partial \phi} &= -\frac{V_1'(k_1^{MX}) - V_2'(k_2^{MX})}{\phi V_1''(k_1^{MX}) + b''(k_1^{MX})}, \\
\frac{\partial k_2^{MX}}{\partial \phi} &= \frac{V_1'(k_1^{MX}) - V_2'(k_2^{MX})}{\phi V_2''(k_2^{MX}) + b''(k_2^{MX})}, \text{ and} \\
\frac{\partial k_2^{MX}}{\partial k_1^{MX}} &= \frac{\phi V_1''(k_1^{MX}) + b''(k_1^{MX})}{\phi V_2''(k_2^{MX}) + b''(k_2^{MX})}.
\end{aligned}$$

Under the assumptions we made about the function V_i's and b, these partial derivatives satisfy

$$\frac{\partial k_1^{MX}}{\partial \phi} > 0, \ \frac{\partial k_2^{MX}}{\partial \phi} < 0, \text{ and } 0 < \frac{\partial k_2^{MX}}{\partial k_1^{MX}} < \infty.$$

Proof of Proposition 4. Since $\frac{\partial K^{MX}}{\partial k_1^{MX}} = 1 + \frac{\partial k_2^{MX}}{\partial k_1^{MX}}$ and $0 < \frac{\partial k_2^{MX}}{\partial k_1^{MX}} < \infty$, we get $1 < \frac{\partial K^{MX}}{\partial k_1^{MX}} < \infty$, which implies $0 < \frac{\partial k_1^{MX}}{\partial K^{MX}} < 1$. And $0 < \frac{\partial k_2^{MX}}{\partial K^{MX}} < 1$ follows from the fact that $\frac{\partial k_2^{MX}}{\partial K^{MX}} = 1 - \frac{\partial k_1^{MX}}{\partial K^{MX}}$.

Proof of Proposition 5. Proposition 4 implies that there is a one-to-one correspondence between K and k_1. So, with the CEO's problem solved in Proposition 1, the owner's problem can be rewritten as finding (S, k_1) such that

$$(S, k_1) = \arg\max_{(\widetilde{S}, \widetilde{k_1})} -\widetilde{S} + (1-\phi)\Phi\left[\widetilde{k}_1, f(\phi, \widetilde{k}_1)\right]$$

subject to

$$\widetilde{S} + \phi\Phi\left[\widetilde{k_1}, f(\phi, \widetilde{k_1})\right] + b(k_1) + b\left[f(\phi, \widetilde{k_1})\right] \geq \mathfrak{C}_1, \tag{59}$$

where $\mathfrak{C}_1 := \overline{u}_C + \overline{u}_{M_1} + \overline{u}_{M_2}$.[30]

The Lagrangian function of the owner's problem is

$$\mathcal{L}(S, k_1, \lambda_3)$$

$$= -S + (1-\phi)\Phi\left[k_1, k_2(\phi, k_1)\right]$$

$$+\lambda_3\left\{S + \phi\Phi\left[k_1, k_2(\phi, k_1)\right] + b(k_1) + b\left[k_2(\phi, k_1)\right] - \mathfrak{C}_1\right\}.$$

The Kuhn-Tucker Conditions are:

$$\mathcal{L}_S = -1 + \lambda_1 \leq 0 \text{ if } S \geq 0 \text{ and } S\mathcal{L}_S = 0 \tag{60}$$

$$\begin{gathered}\mathcal{L}_{k_1} = (1-\phi)\left\{V_1'(k_1) + V_2'\left[k_2(\phi, k_1)\right]\frac{\partial k_2(\phi, k_1)}{\partial k_1} - 1 - \frac{\partial k_2(\phi, k_1)}{\partial k_1}\right\} \\ +\lambda_3\{\phi\left\{V_1'(k_1) + V_2'\left[k_2(\phi, k_1)\right]\tfrac{\partial k_2(\phi,k_1)}{\partial k_1} - 1 - \tfrac{\partial k_2(\phi,k_1)}{\partial k_1}\right\} \\ +b'(k_1) + b'\left[k_2(\phi, k_1)\right]\tfrac{\partial k_2(\phi,k_1)}{\partial k_1}\} \\ \leq 0 \text{ if } k_1 \geq 0 \text{ and } k_1\mathcal{L}_{k_1} = 0\end{gathered}$$

$$\mathcal{L}_{\lambda_3} = S + \phi\Phi\left[k_1, k_2(\phi, k_1)\right] - \mathfrak{C}_1 \geq 0 \text{ if } \lambda_3 \geq 0 \text{ and } \lambda_3\mathcal{L}_{\lambda_3} = 0.$$

Since $S > 0$, the complementary slackness condition of equation (60) implies that $\lambda_1 = 1$, which in turn implies that equation (59) will hold as equality and

$$\mathcal{L}_{k_1} = V_1'(k_1) + b'(k_1) - 1 + \{V_2'\left[k_2(\phi, k_1)\right] + b'\left[k_2(\phi, k_1)\right] - 1\}\frac{\partial k_2(\phi, k_1)}{\partial k_1} = 0, \tag{61}$$

[30] Note here that we can ignore equation (91) since it is redundant given the condition that $\overline{u}_C > 0$ and equation (49).

since we rule out corner solution for k_1 by the assumption that $\lim_{k_i \to 0+} b'(k_i) = \infty$. So, the solution of this case is

$$(S^{MX}, K^{MX}, S_1^{MX}, S_2^{MX}, k_1^{MX}, k_2^{MX}) =$$

$$\{\overline{u}_C - \phi\Phi\left[k_1, k_2(\phi, k_1)+\right] + S_1^{MX} + S_2^{MX}, k_1 + k_2(\phi, k_1),$$

$$\overline{u}_{M_1} - b(k_1), \overline{u}_{M_2} - b(k_2), k_1, k_2(\phi, k_1)\},$$

where k_1 satisfies equation (61).

Proof of Proposition 7. One of the following four mutually exclusive cases must be true: (1) $k_1^{MX} \geq k_1^*$ and $k_2^{MX} \leq k_2^*$, (2) $k_1^{MX} \geq k_1^*$ and $k_2^{MX} > k_2^*$, (3) $k_1^{MX} < k_1^*$ and $k_2^{MX} \leq k_2^*$, and (4) $k_1^{MX} < k_1^*$ and $k_2^{MX} > k_2^*$. Case (1) is not possible because it violates the CEO's decision rule, equation (12) of Proposition 1. Also, cases (3) and (4) are not possible because they violate the owner's decision rule, equation (14) of Proposition 5. Therefore, case (4) must be true, and the wage flattening effect follows directly from Corollary 2.

Proof of Proposition 9. If equation (15) holds, then, by appealing to equation (14), $k_1^{MX} > \widehat{k_1}$ and hence $k_2^{MX} = f(\phi, k_1^{MX}) > f(\phi, \widehat{k_1})$. Therefore, there is overinvestment in total resource available to the firm since $k_1^{MX} + k_2^{MX} > k_1^* + k_2^*$. The proof of the case of underinvestment is symmetric.

Proof of Proposition 10.

$$\begin{aligned}
\frac{dW}{d\phi} &= \left\{V_1'\left[k_1^{MX}(\phi)\right] + b'\left[k_1^{MX}(\phi)\right] - 1\right\}\frac{dk_1^{MX}(\phi)}{d\phi} \\
&+ \left\{V_2'\left[f\left[\phi, k_1^{MX}(\phi)\right]\right] + b'\left[f\left[\phi, k_1^{MX}(\phi)\right]\right] - 1\right\} \\
&* \left(\frac{\partial f\left[\phi, k_1^{MX}(\phi)\right]}{\partial \phi} + \frac{\partial f\left[\phi, k_1^{MX}(\phi)\right]}{\partial k_1^{MX}(\phi)}\frac{dk_1^{MX}(\phi)}{d\phi}\right) \\
&= \{\left\{V_1'\left[k_1^{MX}(\phi)\right] + b'\left[k_1^{MX}(\phi)\right] - 1\right\} \\
&+ \left\{V_2'\left[f\left[\phi, k_1^{MX}(\phi)\right]\right] + b'\left[f\left[\phi, k_1^{MX}(\phi)\right]\right] - 1\right\}\frac{\partial f\left[\phi, k_1^{MX}(\phi)\right]}{\partial k_1^{MX}(\phi)}\}\frac{dk_1^{MX}(\phi)}{d\phi} \\
&+ \left\{V_2'\left[k_2(\phi, k_1^{MX})\right] + b'\left[k_2(\phi, k_1^{MX})\right] - 1\right\}\frac{\partial k_2(\phi, k_1^{MX})}{\partial \phi} \\
&= \left\{V_2'\left[k_2(\phi, k_1^{MX})\right] + b'\left[k_2(\phi, k_1^{MX})\right] - 1\right\}\frac{\partial k_2(\phi, k_1^{MX})}{\partial \phi} \text{ (by Proposition 5)} \\
&> 0. \text{ (since } V_2'\left[k_2(\phi, k_1^{MX})\right] + b'\left[k_2(\phi, k_1^{MX})\right] - 1 < 0 \text{ (by Prop. 4) and } \frac{\partial k_2}{\partial \phi} < 0)
\end{aligned}$$

Proof of Proposition 11. By solving this special case of the model, we obtain the following solutions: First, the first-best resource allocation is

$$k_1^* = \frac{d+b-1}{2(c+a)} + \frac{\alpha}{2(c+a)} \text{ and } k_2^* = \frac{d+b-1}{2(c+a)} - \frac{\alpha}{2(c+a)}.$$

Second, according to the CEO's decision rule in equation (12), the relation between k_1^{MX} and k_2^{MX} is

$$k_2^{MX} = k_1^{MX} - \frac{\phi\alpha}{\phi a + c} \tag{62}$$

and

$$\frac{\partial}{\partial k_1}k_2^{MX} = 1, \frac{\partial}{\partial \phi}k_2^{MX} = -\alpha\frac{c}{(\phi a + c)^2}, \frac{\partial}{\partial \alpha}k_2^{MX} = -\frac{\phi}{\phi a + c} \tag{63}$$

By plugging equation (62) into equation (14) and solving for the decision of the owner, we obtain

$$k_1^{MX} = \frac{b+d-1}{2(c+a)} + \frac{\phi\alpha}{2(\phi a + c)} \text{ and hence } k_2^{MX} = \frac{b+d-1}{2(c+a)} - \frac{\phi\alpha}{2(\phi a + c)}.$$

By the envelope theorem,

$$\begin{aligned} \frac{d\overline{W} - W^{MX}}{d\alpha} &= k_1^* - k_2^* - \left\{ k_1^{MX} - k_2^{MX} + \left[V_2'\left(k_2^{MX}\right) + b'\left(k_2^{MX}\right) - 1 \right] \frac{\partial}{\partial \alpha} k_2^{MX} \right\} \\ &= \frac{\alpha}{(c+a)} - \left[\frac{b+d-1}{2(c+a)} + \frac{\phi\alpha}{2(\phi a + c)} \right] - \frac{b+d-1}{2(c+a)} - \frac{\phi\alpha}{2(\phi a + c)} \\ & - \left\{ -2a\left[\frac{b+d-1}{2(c+a)} - \frac{\phi\alpha}{2(\phi a + c)} \right] + b - 2c\left[\frac{b+d-1}{2(c+a)} - \frac{\phi\alpha}{2(\phi a + c)} \right] + d - 1 \right\} \\ & * \left[-\frac{\phi}{\phi a + c} \right] \\ &= \alpha \frac{\phi^2 a^2 + \phi a c + c^2\left(1 - \phi + \phi^2\right) + ac\phi^2}{(c+a)(\phi a + c)^2} > 0. \end{aligned}$$

Proof of Proposition 12. We will prove this proposition by solving the two-tier principal-agent problem for the multi-segment firm. Note that $k_i^{MN} > 0$ due to the boundary condition in Assumption 2. Furthermore, since equation (26) holds as equality in equilibrium, we have

$$S_i + \phi_{Mi}\Phi(k_1, k_2) = \overline{u}_{Mi} - b(k_i) \text{ for } i \in \{1, 2\}. \quad (64)$$

Also, since the CEO has no incentive to leave any resource not invested, we have

$$k_2 = K - k_1 \quad (65)$$

By plugging equation (64) into equation (21), we have the CEO's problem rewritten as, given S, K, and ϕ_C,

$$\begin{aligned}&(S_1, \phi_{M_1}, k_1, S_2, \phi_{M_2}, k_2)\\ = \;& \arg \max_{(\widetilde{S_1}, \widetilde{\phi_{M_1}}, \widetilde{k_1}, \widetilde{S_2}, \widetilde{\phi_{M_2}}, \widetilde{k_2})} \widetilde{\phi_C}\Phi(\widetilde{k_1}, K - \widetilde{k_1}) + \widetilde{S} + b(\widetilde{k_1}) + b(K - \widetilde{k_1}) - \mathfrak{C}_2\end{aligned}$$

subject to

$$\widetilde{S_1} + \widetilde{S_2} \leq S, \tag{66}$$

$$\widetilde{\phi_{M_1}} + \widetilde{\phi_{M_2}} \leq \phi_C,$$

where $C_2 = \overline{u}_{M_1} + \overline{u}_{M_2}$. The Lagrangian function of the CEO's problem is

$$\begin{aligned}&\mathcal{L}(S_1, S_2, \phi_{M_1}, \phi_{M_2}, k_1, \lambda_1, \lambda_2)\\ &= \phi_C \Phi(k_1, K - k_1) + S + b(k_1) + b(K - k_1) + \mathfrak{C}_2\\ &+\lambda_1[S - S_1 - S_2] + \lambda_2[\phi_C - \phi_{M_1} - \phi_{M_2}].\end{aligned}$$

The Kuhn-Tucker Conditions are:

$$\mathcal{L}_{S_i} = -\lambda_1 \leq 0 \text{ if } S_i \geq 0 \text{ and } S_i \mathcal{L}_{S_i} = 0 \text{ for } i \in \{1, 2\} \tag{67}$$

$$\mathcal{L}_{\phi_{M_1}} = -\lambda_2 \leq 0 \text{ if } \phi_{M_1} \geq 0 \text{ and } \phi_{M_1} \mathcal{L}_{\phi_{M_1}} = 0 \text{ for } i \in \{1, 2\} \tag{68}$$

$$\mathcal{L}_{k_1} = \phi_C[V_1'(k_1) - V_2'(K - k_1)] + b'(k_1) - b'(K - k_1) \leq 0 \text{ if } k_1 \geq 0 \text{ and } k_1 \mathcal{L}_{k_1} = 0 \tag{69}$$

$$\mathcal{L}_{\lambda_1} = S - S_1 - S_2 \geq 0 \text{ if } \lambda_1 \geq 0 \text{ and } \lambda_1 \mathcal{L}_{\lambda_1} = 0 \tag{70}$$

$$\mathcal{L}_{\lambda_2} = \phi_C - \phi_{M_1} - \phi_{M_2} \geq 0 \text{ if } \lambda_2 \geq 0 \text{ and } \lambda_2 \mathcal{L}_{\lambda_2} = 0. \tag{71}$$

We will analyze the following four different possible cases:

Case 1:If $\lambda_1 > 0$ and $\lambda_2 > 0$: We can rule out this case by our assumption that $\overline{u}_C > 0$ since equation (23) will never hold.

Case 2:If $\lambda_1 > 0$ and $\lambda_2 = 0$: In this case, we know that $S_i = 0$ from equation (67) and hence $S = S_1 + S_2 = 0$ from equation (70). Also equation (64) implies that $\phi_{M_1} > 0$ (since we assume that $\overline{u}_{M_i} > b(k_i)$) and hence $\phi_C > 0$ (since $\phi_C > \phi_{M_1}$ from equation (71)). Since $k_1 > 0$, then $L_{k_1} = 0$ implies that

$$\phi_C[V_1'(k_1) - V_2'(K - k_1)] + b'(k_1) - b'(K - k_1) = 0$$

or

$$\phi_C[V_1'(k_1) - V_2'(k_2)] + b'(k_1) - b'(k_2) = 0. \tag{72}$$

By the implicit-function theorem, we can rewrite the relation among k_1, k_2, and ϕ_C as

$$k_2 := g\left(\phi_C, k_1\right) \tag{73}$$

As shown in Proposition 5, for any given ϕ_C, there exists a one-to-one correspondence between k_1 and K. Therefore, we can rewrite the owner's problem as

$$(k_1, \phi_C) = \arg \max_{(\widetilde{k_1}, \widetilde{\phi_C})} (1 - \widetilde{\phi_C})\Phi(\widetilde{k_1}, g\left(\widetilde{\phi_C}, k_1\right)) \tag{74}$$

subject to

$$\widetilde{\phi_C} \leq 1$$

$$\widetilde{\phi_C} \geq \frac{\overline{u}_{M_1} - b(\widetilde{k_1})}{\Phi(\widetilde{k_1}, g\left(\widetilde{\phi_C}, k_1\right))} + \frac{\overline{u}_{M_2} - b(g\left(\widetilde{\phi_C}, k_1\right))}{\Phi(\widetilde{k_1}, g\left(\widetilde{\phi_C}, k_1\right))} \tag{75}$$

$$\widetilde{\phi_C}\Phi(\widetilde{k_1}, g\left(\widetilde{\phi_C}, k_1\right)) + b(\widetilde{k_1}) + b(g\left(\widetilde{\phi_C}, k_1\right) \geq \mathfrak{C}_1, \tag{76}$$

where $C_1 = \overline{u}_C + \overline{u}_{M_1} + \overline{u}_{M_2}$. Note that, with the assumption that $\overline{u}_C > 0$, equation (75) is redundant because it will never bind given the existence of equation (76). So, the Lagrangian function of the owner's problem is

$$\mathcal{L}(k_1, \phi_C, \lambda_1, \lambda_2) = (1-\phi_C)\Phi(k_1, g) + \lambda_3[1-\phi_C] + \lambda_4[\phi_C\Phi(k_1, g) + b(k_1) + b(g) - \mathfrak{C}_1].$$

The Kuhn-Tucker Conditions are:

$$\mathcal{L}_{k_1} = (1-\phi_C)\frac{\partial}{\partial k_1}\Phi + \lambda_4[\phi_C\frac{\partial}{\partial k_1}\Phi + b'(k_1) + b'(g)\frac{\partial}{\partial k_1}g] \leq 0 \text{ if } k_1 \geq 0 \text{ and } k_1\mathcal{L}_{k_1} = 0. \tag{77}$$

$$\mathcal{L}_{\phi_C} = -\Phi + (1-\phi_C)[V_2'(g) - 1]\frac{\partial g}{\partial \phi_C} - \lambda_3$$

$$+\lambda_4\{\Phi + \phi_C[V_2'(g) - 1]\frac{\partial g}{\partial \phi_C} + b'(g)\frac{\partial g}{\partial \phi_C}\} \tag{78}$$

$$\leq 0 \text{ if } \phi_C \geq 0 \text{ and } \phi_C\mathcal{L}_{\phi_C} = 0. \tag{79}$$

$$\mathcal{L}_{\lambda_3} = 1 - \phi_C \geq 0 \text{ if } \lambda_3 \geq 0 \text{ and } \lambda_3\mathcal{L}_{\lambda_3} = 0. \tag{80}$$

$$\mathcal{L}_{\lambda_4} = \phi_C\Phi(k_1, g) + b(k_1) + b(g) - \mathfrak{C}_1 \geq 0 \text{ if } \lambda_4 \geq 0 \text{ and } \lambda_4\mathcal{L}_{\lambda_4} = 0.$$

Case 2.1: When Condition 1 fails ($\lambda_4 > 0$):

Since $k_1 > 0$, the complementary slackness condition in equation (77) implies that

$$(1-\phi_C)\frac{\partial}{\partial k_1}\Phi + \lambda_4[\phi_C\frac{\partial}{\partial k_1}\Phi + b'(k_1) + b'(g)\frac{\partial}{\partial k_1}g] = 0 \tag{82}$$

Note that

$$\frac{\partial}{\partial k_1}\Phi + b'(k_1) + b'(g)\frac{\partial}{\partial k_1}g = 0 \tag{83}$$

since another way of solving the problem is to plug $\phi_C\Phi(k_1, g\ ) + b(k_1) + b(g) = C_1$ into the objective function of the owner's problem, equation (74), and equation (83) follows from the first-order condition. Comparing equations (82) and (83) gives us that

$$\lambda_4 = 1.$$

Plugging this into equation (79) we have

$$\mathcal{L}_{\phi_C} = [V_2'(g) + b'(g) - 1]\frac{\partial g}{\partial \phi_C} - \lambda_3 = 0.$$

This, in turn, implies that $\lambda_3 > 0$ since $[V_2'(g) + b'(g) - 1] < 0$ from Proposition 7 and $\frac{\partial g}{\partial \phi_C} = \frac{V_1'(k_1^{MN}) - V_2'(g)}{\phi V_2''(g) + b''(g)} < 0$. Therefore, we know that equation (22) will bind and hence $\phi_C = 1$.

Case 2.2: When Condition 1 fails ($\lambda_4 = 0$):

Since $\phi_C > 0$, then the complementary slackness condition in equation (79) implies that

$$1 - \frac{\Phi + \lambda_3}{[V_2'(g) - 1]\frac{\partial g}{\partial \phi_C}} = \phi_C \Leftrightarrow$$

$$\phi_C = \frac{[V_2'(g)-1]V_1'(k_1) - (\Phi+\lambda_3)\, b''(g)}{[V_2'(g)-1]V_1'(k_1) + (\Phi+\lambda_3)\, V_2''(g)}. \tag{84}$$

If $\lambda_3 > 0$, then $\phi_C = 1$ (from equation (80)), which contradict with equation (84). Therefore $\lambda_3 = 0$. When Condition 2 also holds, i.e.

$$[V_2'(g)-1]V_1'(k_1) - \Phi b''(g) < 0, \tag{85}$$

we know that

$$[V_2'(g)-1]V_1'(k_1) + \Phi V_2''(g) < 0 \text{ because} \tag{86}$$

$$[V_2'(g)-1]V_1'(k_1) + \Phi V_2''(g) < [V_2'(g)-1]V_1'(k_1) - \Phi b''(g) \tag{87}$$

(Recall that $b''(g)$ and $V_2''(g)$ are both negative.). Equations (85), (86), and (87) imply that $1 > \phi_C > 0$. Therefore, equation (84) is an interior solution.

Also, in this case, equation (77) becomes

$$\mathcal{L}_{k_1} = V_1'(k_1) + V_2'(g)\frac{\partial}{\partial k_1}g - 1 - \frac{\partial}{\partial k_1}g \leq 0 \text{ if } k_1 \geq 0 \text{ and } k_1\mathcal{L}_{k_1} = 0.$$

If $k_1 > 0$, then

$$V_1'(k_1) + V_2'(g(k_1))\frac{\partial g}{\partial k_1} - 1 - \frac{\partial g}{\partial k_1} = 0 \tag{88}$$

So the solution will be

$$(S, K, \phi_C, S_1, S_2, \phi_{M_1}, \phi_{M_2}, k_1)$$
$$= (0, k_1+g, \frac{[V_2'(g)-1]V_1'(k_1) - \Phi b''(g)}{[V_2'(g)-1]V_1'(k_1) + \Phi V_2''(g)}, 0, 0, \frac{\overline{u}_{M_1} - b(k_1)}{\Phi(k_1, g)}, \frac{\overline{u}_{M_2} - b\,(g)}{\Phi\,(k_1, g)}, k_1),$$

where k_1 satisfies equation (88).

As of when Condition 2 fails, i.e.

$$[V_2'(g) - 1]V_1'(k_1) - \Phi b''(g) \geq 0,$$

if $[V_2'(g)-1]V_1'(k_1)+\Phi V_2''(g) < 0$, then.$\phi_C \leq 0$; else if $[V_2'(g)-1]V_1'(k_1)+\Phi V_2''(g) \geq 0$, then $\phi_C > 1$. Therefore, $\lambda_4 = 0$ cannot be the case when Condition 2 fails.

Note that Condition 1 is only consistent with the cases when $S = 0$ because otherwise the owner can always gain from reducing S a infinitesimal amount without violating the CEO's participation constraint. Therefore, the following two cases are associated with the failure of Condition 1.

Case 3:If $\lambda_1 = 0$ and $\lambda_2 > 0$: In this case, we know that $\phi_{M_1} = 0$ from equation (68) and hence $\phi_C = \phi_{M_1} + \phi_{M_2} = 0$ from equation (71). Also equation (64) implies that $S_i > 0$ (since we assume that $\overline{u}_{M_i} > b(k_i)$) and hence $S_C > 0$ (since $S_C > S_i$ from equation (70)). Since the CEO has no incentive at all ($\phi_C = 0$) to allocate the resource to increase segmental productivity, he will just split equally whatever he get from the owner to each segment managers, i.e. $k_1^{MN} = k_2^{MN}$. However, this case cannot be a equilibrium because the owner can always gain by substituting some of the cash paid to the CEO with shares that has the same value, resulting in better resource allocations (according to Proposition 10) and hence higher firm value without breaking the participation constraint.

Case 4:If $\lambda_1 = 0$ and $\lambda_2 = 0$, the solution of k_1 will be the same as in case 2. The

Lagrangian function of the owner's problem is

$$\mathcal{L}(S, k_1, \phi_C, \lambda_1, \lambda_2) = -S + (1-\phi_C)\Phi(k_1, k_2) + \lambda_5[1-\phi_C]$$
$$+\lambda_6[\phi_C \Phi(k_1, g) + S + b(k_1) + b(g) - \mathfrak{C}_1].$$

The Kuhn-Tucker Conditions are:

$$\mathcal{L}_S = -1 + \lambda_6 \leq 0 \text{ if } S \geq 0 \text{ and } S\mathcal{L}_S = 0. \tag{89}$$

$$\mathcal{L}_{k_1} = (1-\phi_C)\frac{\partial}{\partial k_1}\Phi + \lambda_6[\phi_C \frac{\partial}{\partial k_1}\Phi + b'(k_1) + b'(g)\frac{\partial}{\partial k_2}k_2] \leq 0 \text{ if } k_1 \geq 0 \text{ and } k_1\mathcal{L}_{k_1} = 0. \tag{90}$$

$$\mathcal{L}_{\phi_C} = -\Phi + (1-\phi_C)[V_2'(k_2) - 1]\frac{\partial k_2}{\partial \phi_C} - \lambda_5$$
$$+\lambda_6\{\Phi + \phi_C[V_2'(k_2) - 1]\frac{\partial k_2}{\partial \phi_C} + b'(k_2)\frac{\partial k_2}{\partial \phi_C}\}$$
$$\leq 0 \text{ if } \phi_C \geq 0 \text{ and } \phi_C \mathcal{L}_{\phi_C} = 0.$$

$$\mathcal{L}_{\lambda_5} = 1 - \phi_C \geq 0 \text{ if } \lambda_5 \geq 0 \text{ and } \lambda_5 \mathcal{L}_{\lambda_5} = 0.$$

$$\mathcal{L}_{\lambda_6} = \phi_C \Phi(k_1, g) + S + b(k_1) + b(g) - \mathfrak{C}_1 \geq 0 \text{ if } \lambda_6 \geq 0 \text{ and } \lambda_6 \mathcal{L}_{\lambda_6} = 0.$$

Since $S > 0$, equation (89) implies that $\lambda_6 = 1$. However, for the same reason as in case 2.1, $\phi_C = 1$.

To sum up, when conditions 1 and 2 hold (as in case 2.2), $0 < \phi_C < 1$ and the solution is as stated above, otherwise (as in case 2.1 or 4), $\phi_C = 1$.

Proof of Proposition 16. (a) The proof is similar to the one of Proposition 7. The only difference is when managerial ownership is endogenous the owner's decision rule change from equation (14) to (31). The listed three cases are the the ones that are consistent with the CEO's and the owner's first order conditions.

(b) For each segment i, we know

$$\begin{aligned} V_i\left(k_i^{SN}\right)+b\left(k_i^{SN}\right)-k_i^{SN} &= V_i\left(k_i^*\right)+b\left(k_i^*\right)-k_i^* > \\ V_i\left(k_i^-\right)+b\left(k_i^-\right)-k_i^- &> V_i\left(k_i^-\right)-k_i^- > V_i\left(k_i^{MN}\right)-k_i^{MN}. \end{aligned}$$

So

$$\sum_{i=1}^{2}\left[V_i\left(k_i^{SN}\right)+b\left(k_i^{SN}\right)-k_i^{SN}\right] > \sum_{i=1}^{2} V_i\left(k_i^{MN}\right)-k_i^{MN}.$$

Proof of Proposition 17. The proof is similar to the one for Proposition 9.

Proof of Proposition 19. By plugging equation (62) into equation (31) and solving for the decision of the owner, we obtain

$$k_1^{MN}=\frac{b-1}{2a}+\frac{\phi\alpha}{2\left(\phi a+c\right)} \text{ and hence } k_2^{MN}=\frac{b-1}{2a}-\frac{\phi\alpha}{2\left(\phi a+c\right)}.$$

By envelop theorem, we have

$$\begin{aligned} \frac{d\overline{W}-W^{MX}}{d\alpha} &= k_1^*-k_2^*-\left\{k_1^{MN}-k_2^{MN}+\left[V_2'\left(k_2^{MN}\right)-1\right]\frac{\partial}{\partial\alpha}k_2^{MN}\right\} \\ &= \frac{1}{2}\alpha\frac{\phi^2a^2+\phi ac+\phi ac-\phi^2ac+2c^2-2\phi c^2}{\left(c+a\right)\left(\phi a+c\right)^2} > 0. \end{aligned}$$

5 Appendix B

Proof of Lemma 20. With (33) and (35) binding under Assumption 5 and (34) binding naturally, the three first-order conditions follow directly.

Proof of Lemma 21. With (36), (39) and (42) binding under Assumption 5 and (38) and (41) binding naturally, the three first-order conditions follow directly.

Proof of Proposition 22. By applying *the implicit-function rule* to (46), we have

$$\frac{\partial k_m}{\partial k_n} = \frac{\rho\phi V_n''(k_n) + b''(k_n)}{\rho\phi V_m''(k_m) + b''(k_m)} > 0,$$

and also that

$$\frac{\partial k_m}{\partial \rho} = -\frac{\phi\left[V_m'(k_m) - V_n'(k_n)\right]}{\rho\phi V_m''(k_m) + b''(k_m)} > 0 \text{ and } \frac{\partial k_n}{\partial \rho} = \frac{\phi\left[V_m'(k_m) - V_n'(k_n)\right]}{\rho\phi V_n''(k_n) + b''(k_n)} < 0.$$
$$\frac{\partial k_m}{\partial \phi} = -\frac{\rho\left[V_m'(k_m) - V_n'(k_n)\right]}{\rho\phi V_m''(k_m) + b''(k_m)} > 0 \text{ and } \frac{\partial k_n}{\partial \phi} = \frac{\rho\left[V_m'(k_m) - V_n'(k_n)\right]}{\rho\phi V_n''(k_n) + b''(k_n)} < 0.$$

Again By applying the implicit-function rule to (47), we have

$$\frac{\partial k_n}{\partial k_l} = \frac{\left[\phi V_l''\left(k_l\right) + b''\left(k_l\right)\right]\left(\frac{\partial k_m}{\partial k_n} + 1\right)}{\frac{\partial}{\partial k_n} MB^C} \tag{91}$$

Also, under (49), we have, via (91),

$$\frac{\partial k_n}{\partial k_l} > 0 \text{ and hence } \frac{\partial k_m}{\partial k_l} = \frac{\partial k_n}{\partial k_l}\frac{\partial k_m}{\partial k_n} > 0.$$

Proof of Proposition 23. First, $K = k_l + k_D$ implies

$$\frac{\partial K}{\partial k_l} = 1 + \frac{\partial k_D}{\partial k_l} \tag{92}$$

. Second,

$$0 < \frac{\partial k_D}{\partial k_l} = \frac{\partial k_n}{\partial k_l} + \frac{\partial k_n}{\partial k_l}\frac{\partial k_m}{\partial k_n} < \infty. \tag{93}$$

Third, (92) and (93) implies that $1 < \frac{\partial K}{\partial k_l} < \infty$, which in turn implies $0 < \frac{\partial k_l}{\partial K} < 1$. Finally, $0 < \frac{\partial k_D}{\partial K} < 1$ follows from that fact that $\frac{\partial k_D}{\partial K} = 1 - \frac{\partial k_l}{\partial K}$. This proves the first two inequalities. The last two inequality can be proved by the same procedure applied to $k_D = k_m + k_n$.

We need the following lemma for the proof of Proposition 24.

Lemma 34 *Inside the division, the resource allocated to the more profitable segment is always greater than the resource allocated to the less profitable segment, i.e.* $k_m > k_n$.

Proof of Lemma 34. It follows directly from (46) since by definition $n > m$.

Proof of Proposition 24. The following are the eight possible cases:

Case 1: $k_1 < k_1^*$, $k_2 < k_2^*$, and $k_3 < k_3^*$: This cases is impossible because (48) will not hold since $\frac{\partial k_m}{\partial k_l} > 0$ and $\frac{\partial k_n}{\partial k_l} > 0$ under Assumption 6.

Case 2: $k_1 \geq k_1^*$, $k_2 \geq k_2^*$, and $k_3 \geq k_3^*$: This cases is impossible for the same reason as in Case 1.

Case 3: $k_1 < k_1^*$, $k_2 < k_2^*$, and $k_3 \geq k_3^*$: It is possible that all the I.C. constraints are satisfied.

Case 4: $k_1 < k_1^*$, $k_2 \geq k_2^*$, and $k_3 \geq k_3^*$: It is possible that all the I.C. constraints are satisfied.

Case 5: $k_1 < k_1^*$, $k_2 \geq k_2^*$, and $k_3 < k_3^*$: This cases is impossible for $\mathcal{F}$ because $V_2'(k_2) + b'(k_2) < 1$ and $V_3'(k_3) + b'(k_3) \geq 1$ implies $V_2'(k_2) - V_3'(k_3) + b'(k_2) - b'(k_3) < 0$ which implies $\phi[V_2'(k_2) - V_3'(k_3)] + b'(k_2) - b'(k_3) < 0$ which violate equations (77) and (44). $\mathcal{T}_1$ can not be the case because (46) will be violated. $\mathcal{T}_2$ can not be the case for the following reason: First,

$$\rho\phi\left[V_1'(k_1) - V_3'(k_3)\right] + \left[b'(k_1) - b'(k_3)\right] = 0 \text{ ((46)) and } 1 \geq \rho$$

imply that

$$\phi\left[V_1'(k_1) - V_3'(k_3)\right] + \left[b'(k_1) - b'(k_3)\right] > 0. \tag{94}$$

Second, $V_2'(k_2) + b'(k_2) < 1$ and $V_3'(k_3) + b'(k_3) \geq 1$ imply $\left[V_3'(k_3) - V_2'(k_2)\right] + \left[b'(k_3) - b'(k_2)\right] > 0$ and hence

$$\phi\left[V_3'(k_3) - V_2'(k_2)\right] + \left[b'(k_3) - b'(k_2)\right] > 0, \tag{95}$$

since $k_3 < k_2$. Third, equations (94) and (95) imply that

$$\phi\left[V_1'(k_1) - V_2'(k_2)\right] + \left[b'(k_1) - b'(k_2)\right] > 0. \tag{96}$$

Finally, equations (94) and (96) imply that

$$\begin{aligned}&\left\{\phi\left[V_1'(k_1) - V_2'(k_2)\right] + \left[b'(k_1) - b'(k_2)\right]\right\}\frac{\partial k_1}{\partial k_3}\\&+\phi\left[V_3'(k_3) - V_2'(k_2)\right] + \left[b'(k_3) - b'(k_2)\right] > 0,\end{aligned}$$

which violates (47). Only under $\mathcal{T}_3$, it is possible that all the I.C. constraints are satisfied.

Case 6: $k_1 \geq k_1^*$, $k_2 < k_2^*$, and $k_3 \geq k_3^*$: This cases is impossible for $\mathcal{F}$ because $V_1'(k_1) + b'(k_1) < 1$ and $V_2'(k_2) + b'(k_2) \geq 1$ implies $V_1'(k_1) - V_2'(k_2) + b'(k_1) - b'(k_2) < 0$ which implies $\phi\left[V_1'(k_1) - V_2'(k_2)\right] + b'(k_1) - b'(k_2) < 0$ which violate equations (77) and (44). $\mathcal{T}_3$ can not be the case because (46) will be violated. $\mathcal{T}_2$ can not be the case for the following reason: First,

$$\rho\phi\left[V_1'\left(k_1\right)-V_3'\left(k_3\right)\right]+\left[b'\left(k_1\right)-b'\left(k_3\right)\right]=0 \text{ ((46)) and } 1\geq\rho$$

imply that

$$\phi\left[V_1'\left(k_1\right)-V_3'\left(k_3\right)\right]+\left[b'\left(k_1\right)-b'\left(k_3\right)\right]>0.$$

Second, $V_1'\left(k_2\right)+b'\left(k_2\right)\geq 1$ and $V_1'\left(k_1\right)+b'\left(k_1\right)<1$ imply $\left[V_1'\left(k_1\right)-V_2'\left(k_2\right)\right]+\left[b'\left(k_1\right)-b'\left(k_2\right)\right]<0$ and hence

$$\phi\left[V_1'\left(k_1\right)-V_2'\left(k_2\right)\right]+\left[b'\left(k_1\right)-b'\left(k_2\right)\right]<0.$$

since $k_2<k_1$. Third, equations (94) and (95) imply that

$$\phi\left[V_3'\left(k_1\right)-V_2'\left(k_2\right)\right]+\left[b'\left(k_3\right)-b'\left(k_2\right)\right]<0.$$

Finally, equations (94) and (96) imply that

$$\begin{aligned}&\left\{\phi\left[V_1'\left(k_1\right)-V_2'\left(k_2\right)\right]+\left[b'\left(k_1\right)-b'\left(k_2\right)\right]\right\}\frac{\partial k_1}{\partial k_3}\\&+\phi\left[V_3'\left(k_3\right)-V_2'\left(k_2\right)\right]+\left[b'\left(k_3\right)-b'\left(k_2\right)\right]<0,\end{aligned}$$

which violates (47). Only under $\mathcal{T}_1$, it is possible that all the I.C. constraints are satisfied.

Case 7: $k_1\geq k_1^*$, $k_2<k_2^*$, and $k_3<k_3^*$: This cases is impossible for $\mathcal{F}$ because $V_1'\left(k_1\right)+b'\left(k_1\right)<1$ and $V_3'\left(k_3\right)+b'\left(k_3\right)\geq 1$ implies $V_1'\left(k_1\right)-V_3'\left(k_3\right)+b'\left(k_1\right)-b'\left(k_3\right)<0$ which implies $\phi\left[V_1'\left(k_1\right)-V_3'\left(k_3\right)\right]+b'\left(k_1\right)-b'\left(k_3\right)<0$ which violate equations (77) and (44). It can not be $\mathcal{T}_3$ or $\mathcal{T}_2$ because (46) will be violated. It can not be $\mathcal{T}_1$ because $V_1'\left(k_1\right)+b'\left(k_1\right)<1$, $V_1'\left(k_2\right)+b'\left(k_2\right)\geq 1$ and $V_3'\left(k_3\right)+b'\left(k_3\right)\geq 1$

imply

$$
\begin{aligned}
\left[V_2'(k_2) - V_1'(k_1)\right] + \left[b'(k_2) - b'(k_1)\right] &> 0 \text{ and hence (since } k_1 > k_2\text{)} \\
\phi\left[V_2'(k_2) - V_1'(k_1)\right] + \left[b'(k_2) - b'(k_1)\right] &> 0; \\
\left[V_3'(k_3) - V_1'(k_1)\right] + \left[b'(k_3) - b'(k_1)\right] &> 0 \text{ and hence (since } k_1 > k_3\text{)} \\
\phi\left[V_3'(k_3) - V_1'(k_1)\right] + \left[b'(k_3) - b'(k_1)\right] &> 0,
\end{aligned}
$$

which imply

$$
\begin{aligned}
&\left\{\phi\left[V_2'(k_2) - V_1'(k_1)\right] + \left[b'(k_2) - b'(k_1)\right]\right\}\frac{\partial k_1}{\partial k_3} \\
&+\phi\left[V_3'(k_3) - V_1'(k_1)\right] + \left[b'(k_3) - b'(k_1)\right] > 0,
\end{aligned}
$$

which violates (47).

Case 8: $k_1 \geq k_1^*$, $k_2 \geq k_2^*$, and $k_3 < k_3^*$: This cases is impossible for $\mathcal{F}$ because $V_1'(k_1) + b'(k_1) < 1$ and $V_3'(k_3) + b'(k_3) \geq 1$ implies $V_1'(k_1) - V_3'(k_3) + b'(k_1) - b'(k_3) < 0$ which implies $\phi\left[V_1'(k_1) - V_3'(k_3)\right] + b'(k_1) - b'(k_3) < 0$ which violates equations (77) and (44). It can not be $\mathcal{T}_1$ or $\mathcal{T}_2$ because (46) will be violated. It can not be $\mathcal{T}_3$ because $V_1'(k_1) + b'(k_1) < 1$, $V_1'(k_2) + b'(k_2) < 1$ and $V_3'(k_3) + b'(k_3) \geq 1$ imply

$$
\begin{aligned}
\left[V_1'(k_1) - V_3'(k_3)\right] + \left[b'(k_1) - b'(k_3)\right] &< 0 \\
\text{and hence } \phi\left[V_1'(k_1) - V_3'(k_3)\right] + \left[b'(k_1) - b'(k_3)\right] &< 0; \\
\left[V_2'(k_2) - V_3'(k_3)\right] + \left[b'(k_2) - b'(k_3)\right] &< 0 \\
\text{and hence } \phi\left[V_2'(k_2) - V_3'(k_3)\right] + \left[b'(k_2) - b'(k_3)\right] &< 0,
\end{aligned}
$$

which imply

$$\left\{\phi\left[V_1'\left(k_1\right)-V_3'\left(k_3\right)\right]+\left[b'\left(k_1\right)-b'\left(k_3\right)\right]\right\}\frac{\partial k_1}{\partial k_3}$$
$$+\phi\left[V_2'\left(k_2\right)-V_3'\left(k_3\right)\right]+\left[b'\left(k_2\right)-b'\left(k_3\right)\right]<0,$$

which violates (47).

To sum up, only under $\mathcal{T}_3$ can case 5 (segment 3 is underinvested) be possible, and only under $\mathcal{T}_1$ can case 6 (segment 1 is overinvested) be possible. This concludes the proof.

Proof of Proposition 25. (a) The result directly follows from Proposition 23.

(b) $\mathcal{T}_1$: Suppose not, then $V_1'\left(k_1\right)+b'\left(k_1\right)\geq 1$ and $V_2'\left(k_2\right)+b'\left(k_2\right)<1$ implies that

$$\left[V_2'\left(k_2\right)-V_1'\left(k_1\right)\right]+b'\left(k_2\right)-b'\left(k_1\right)<0$$

which in turn implies there exist $\overline{\phi}$ such that

$$\phi\left[V_2'\left(k_2\right)-V_1'\left(k_1\right)\right]+b'\left(k_2\right)-b'\left(k_1\right)<0 \text{ for } \phi>\overline{\phi}. \tag{97}$$

Also,

$$\rho\phi\left[V_2'(k_2)-V_3'(k_3)\right]+b'(k_2)-b'(k_3)=0$$

implies that

$$\phi V_2'(k_2)+b'(k_2)>\phi V_3'(k_3)+b'(k_3). \tag{98}$$

Combining (97) and (98) obtains

$$\frac{\partial k_2}{\partial k_3}\left[\phi V_2'\left(k_2\right)-\phi V_1'\left(k_1\right)+b'\left(k_2\right)-b'\left(k_1\right)\right]$$
$$+\phi V_3'\left(k_3\right)-\phi V_1'\left(k_1\right)+b'\left(k_3\right)-b'\left(k_1\right)<0$$

which violates (47).

The proof for $\mathcal{T}_3$ is symmetric.

Proof of Proposition 26. For $\mathcal{F}$,

$$\phi\left[V_2'(k_2) - V_1'(k_1)\right] + b'(k_2) - b'(k_1) = 0$$

$$\phi\left[V_3'(k_3) - V_1'(k_1)\right] + b'(k_3) - b'(k_1) = 0$$

When $\rho = 1$. For $\mathcal{T}_l$, we have

$$\phi\left[V_m'(k_m) - V_n'(k_n)\right] + b'(k_m) - b'(k_n) = 0 \text{ and} \tag{99}$$

$$\begin{aligned}\phi&\left[V_m'\left(k_m\right)\frac{\partial k_m}{\partial k_n} + V_n'\left(k_n\right) - V_l'\left(k_l\right)\left(\frac{\partial k_m}{\partial k_n}+1\right)\right]\\ &+b'\left(k_m\right)\frac{\partial k_m}{\partial k_n} + b'\left(k_n\right) - b'\left(k_l\right)\left(\frac{\partial k_m}{\partial k_n}+1\right) = 0\end{aligned} \tag{100}$$

Equations (99) and (100) implies

$$\phi\left[V_l'(k_l) - V_n'(k_n)\right] + b'(k_l) - b'(k_n) = 0.$$

In both cases,

$$\phi V_1'(k_1) + b'(k_1) = \phi V_2'(k_2) + b'(k_2) = \phi V_3'(k_3) + b'(k_3),$$

i.e. the relationship between k_1, k_2, and k_3 are the same. This implies that O's aggregate investment will also be the same. Therefore $W^{\mathcal{F}} = W^{\mathcal{T}_l}$ when $\rho = 1$.

Proof of Proposition 27. (a). For $\mathcal{T}_1$, lets write k_2 as a function of k_3 and then k_3

as a function of k_1. As a result,

$$
\begin{aligned}
\frac{\partial W^{\mathcal{T}_1}(\phi,\rho)}{\partial \phi} &= \left[V_2'(k_2)+b'(k_2)-1\right]\left[\frac{\partial k_2}{\partial \phi}+\frac{\partial k_2}{\partial k_3}\frac{\partial k_3}{\partial \phi}+\frac{\partial k_2}{\partial k_3}\frac{\partial k_3}{\partial k_1}\frac{\partial k_1}{\partial \phi}\right] \\
&\quad +\left[V_3'(k_3)+b'(k_3)-1\right]\left[\frac{\partial k_3}{\partial \phi}+\frac{\partial k_3}{\partial k_1}\frac{\partial k_1}{\partial \phi}\right] \\
&\quad +\left[V_1'(k_1)+b'(k_1)-1\right]\frac{\partial k_1}{\partial \phi} \\
&= \left[V_2'(k_2)+b'(k_2)-1\right]\left[\frac{\partial k_2}{\partial \phi}+\frac{\partial k_2}{\partial k_3}\frac{\partial k_3}{\partial \phi}\right] \\
&\quad +\left[V_3'(k_3)+b'(k_3)-1\right]\frac{\partial k_3}{\partial \phi} \text{ (by the envelop theorem)} \\
&= \left[V_3'(k_3)+b'(k_3)-1\right]\frac{\partial k_3}{\partial \phi} \text{ (since } \frac{\partial k_2}{\partial \phi}+\frac{\partial k_2}{\partial k_3}\frac{\partial k_3}{\partial \phi}=0\text{)} \\
&> 0 \text{ (since } V_3'(k_3)+b'(k_3)-1>0 \text{ and } \frac{\partial k_3}{\partial \phi}<0\text{).}
\end{aligned}
$$

The proof for $\mathcal{T}_2$ and $\mathcal{T}_3$ is similar with appropriate rewriting the relationship between k_1, k_2, and k_3.

(b). The proof for $\partial W^{\mathcal{T}_l}(\phi,\rho)/\partial\rho > 0$ is the same as the case of (a). $\mathcal{F}$ is more efficient than $\mathcal{T}_l$ follows from $\partial W^{\mathcal{T}_l}(\phi,\rho)/\partial\rho > 0$ and Proposition 26.

6 References

[1] A. W. Beggs, Queues and Hierarchies, *Review of Economic Studies*, **68** 2001, 297-322.

[2] R. Antel and G. Eppen, Capital Rationing and Organizational Slack in Capital Budgeting, *Management Science* **31** 1985, 163-174.

[3] P. Berger and E. Ofek, Diversification's effect on firm value, *Journal of Financial Economics*, **37** 1995, 39-65.

[4] A. Bernardo, H. Cai and J. Luo, Capital Budgeting Compensation with Asymmetric Information and Moral Hazard, *Journal of Financial Economics*, **61** 2001, 311-344.

[5] R. Bliss and R. Rosen, CEO Compensation and Bank Mergers, *Journal of Financial Economics*, **61** 2001, 107-138.

[6] P. Bolton and M. Dewatripont, The Firm as a Communication Network, *Quarterly Journal of Economics*, **109** (4) 1994, 809-839.

[7] P. Bolton and D. Scharfstein, Corporate Finance, the Theory of the Firm, and Organization, *Journal of Economic Perspectives*, **12** 1998, 95-114.

[8] M. Buble, Transition Processes of Organizational Structures in Large Enterprises, *Management*, **1** (1), 1996, 1-16.

[9] G. Calvo and S. Wellisz, Supervision, Loss of Control and the Optimal Size of the Firm, *Journal of Political Economy*, **86** 1978, 943-952.

[10] S. Grossman and O. Hart, The Costs and Benefits of Ownership: A Theory of Vertical and Lateral Integration, *Journal of Political Economy*, **94** 1986, 691-719.

[11] M. Hammer and J. Champy, Reengineering the Corporation, New York: Harper Collins, 1993.

[12] O. Hart and J. Moore, Property Rights and the Nature of the Firm, *Journal of Political Economy*, **98** 1990, 1119-58.

[13] O. Hart and J. Moore, On the Design of Hierarchies: Coordination and Specialization, NEBR working paper 7388, 1999.

[14] O. Hart and J. Moore, Default and Renegotiation: A Dynamic Model of Debt, *Quarterly Journal of Economics*, **113** 1998, 1-41.

[15] M. Harris and A. Raviv, The Capital Budgeting Process: Incentives and Information, *Journal of Finance*, **51** (4) 1996, 1139-1174.

[16] M. Harris and A. Raviv, Capital Budgeting and Delegation, *Journal of Financial Economics*, **50** 1998, 259-289.

[17] R. Inderst and H. Müller, Project Bundling, Liquidity Spillovers, and Capital Market Discipline, Discussion Paper, University of Mannheim, 2000.

[18] R. Inderst, H. Müller, and K. Wärneryd, Influence Costs and Hierarchy, Discussion Paper, University of Mannheim, 2000.

[19] M. Jensen and W. Mercklin, Theory of the Firm: Managerial Behavior, Agency Costs, and Ownership Structure, *Journal of Financial Economics,* **3** 1979, 305-360.

[20] L. Lang and R. Stulz, Tobin's *q*, Corporation Diversification, and Firm Performance, *Journal of Political Economy,* **102** 1994, 1248-1280.

[21] E. Maskin, Y. Quian, and C. Xu, Incentives, Information, and Organizational Form, *Review of Economic Studies*, **67** 2000, 359-378.

[22] J. A. Mirrless, The Optimal Structure of Incentives and Authority within an Organization, *Bell Journal of Economics,* **7** 1976, 105-131.

[23] R. Morck, A. Shleifer, and R. Vishny, Do Managerial Objectives Drives Bad Acquisitions?, *Journal of Finance,* **45** 1990, 31-48.

[24] R. Rajan, H. Servaes, and L. Zingales, The Cost of Diversity: The Diversification Discount and Inefficient Investment, *Journal of Finance*, **55** (1) 2000, 35-80.

[25] R. Rajan and L. Zingales, The Firm as a Dedicated: A theory of the Origins and Growth of Firms, *Quarterly Journal of Economics,* **116** (3) 2001, 805-851.

[26] D. Scharfstein, The Dark Side of the Internal Capital market II: Evidence from Diversified Conglomerate, NEBR working paper 6352, 1998.

[27] D. Scharfstein and J. Stein, The Dark Side of the internal capital market: Divisional rent seeking and inefficient investment, *Journal of Finance,* **55** 2000, 2537-64.

[28] H. Shin and R. Stulz, Are the internal capital market Efficient?, *Quarterly Journal of Economics* **113** 1998, 531-553.

[29] J. Stein, the Internal Capital Market and the Competition for Corporate Resources, *Journal of Finance*, **52** 1997, 111-134.

[30] L. Stole and J. Zwiebel, Organizational Design and Technology Choice under Intrafirm Bargaining, *American Economic Review*, **86** (1) 1996, 195-222.

[31] R. Stulz, Managerial Discretion and Optimal Financing Policies, *Journal of Financial Economics*, **26** 1991, 3-27.

[32] T. Whited, Is It Inefficient Investment that Causes the Diversification Discount?, *Journal of Finance*, forthcoming.

[33] O. Williamson, *Markets and Hierarchies: Analysis and Antitrust Implications*, New York: Free Press, 1975.

[34] J. Wulf, Influence and Inefficiency in the Internal Capital Market: Theory and Evidence, Working Paper, University of Pennsylvania, 2000.

[35] O. Williamson, Hierarchical Control and Optimal Firm Size, *Journal of Political Economy*, **75** (2) 1967, 123-138.

[36] L. Zingales, In Search of New Foundations, *Journal of Finance*, **55** 2000, 1623-53.