Market for Degrees and Educational Standards*

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Abstract

This paper interprets, in an equilibrium model, the typically observed allocations of good students in top colleges and weaker ones in the others. Our theory is that modulo capacity constraints, weaker students do not end up in top, more expensive colleges because for them it is not worth it, in the sense that their incremental benefit from attending harder higher-standard colleges is lower than higher cost they would have to bear. We thus interrelate educational standards and learning ability, obtaining that universities that teach deeper material provide higher market value to better students, but not to less able students. This is in contrast to the more traditional view that students are attracted to more selective universities because they will find higher quality peers there. The model features stratification by ability, which is due to matching between teaching standards and learning potential, and provides foundation for the "mismatch" debate concerning affirmative action in selective universities. Our analysis reveals how subsidies may reduce welfare by distorting students allocation in the various types of degrees.

1 Introduction

What happens to a university if it offers easy courses and gives everybody an A? What kind of students will it get? We all know the answer: the worst students. It is implicit that the answer is really that it gets the worst students *in equilibrium*. This paper makes this qualification explicit. Of course once the equilibrium approach is taken one simultaneously has to answer the specular question of why good students

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want to go to the more selective colleges. The symmetric answer, the one we subscribe to, is that at more selective universities students can learn deeper material, which has greater market value - but only for those with the ability to appreciate the greater depth.

This view is, somewhat surprisingly, not widely shared. A more common approach is to assume that a university's quality depends on the average ability of its student body, via peer effects (see for example Epple-Romano (1998), Epple-Romano-Sieg (2006), Dale-Krueger (2002, 2011) or MacLeod-Urquiola (2011)): good students go to selective colleges because other good students go there. This view has distant roots. In this vein Dale-Krueger (2002) quote Hunt (1963) as saying that "The C student from Princeton earns more than the A student from Podunk not mainly because he has the prestige of a Princeton degree, but merely because he is abler. The golden touch is possessed not by the Ivy League College, but by its students." We disagree with this view. Peer effects are surely relevant (see Epple-Romano (2011)), but if the Princeton student body were transported to a college in Zimbabwe we doubt they would leave with the same market value as they have when they leave Princeton. We think the golden touch is possessed by the good universities, more precisely by their teachers: it is they who transmit deeper, more useful knowledge. The driving force is the standard to which the students are taught. Top colleges want and get more able students because they teach to the highest standards and good students are the ones who get best value from those standards.

In the end the issue boils down to what one has in mind for learning technology. If one thinks that peer effects are dominant then appropriate equilibrium models, student sorting and policy consequences are those in the literature just cited. The model presented in this paper seems more appropriate if, on the other hand, one gives more weight to individual acquired competence and to the fact that the ability to learn from a higher quality institution may decline after a peak if the student is not sufficiently well equipped. Evidence on this so-called "mismatch", as reported notably in the recent book by Sander-Taylor (2012), appears to be strong. Focusing specifically on affirmative action (which we do not discuss), they reach the conclusion that placing weaker students in strong educational environments impairs their learning progress so dramatically that preferences end up hurting underrepresented minorities far more than they help them.[1]

[1]The book is an outgrow of Sander's earlier much quoted and much discussed paper Sander (2004), going on to the more recent Sander (2011). Of course the opposite view is also present is

This paper then develops a simple model where a student's market value derives from learning, more learning generating higher value. Learning in turn depends on the depth of material covered in courses, which we call the educational standard, and on students ability. Students are heterogeneous in their ability, and more able students gain more value from a higher standard of teaching than less able students. Universities differ in setting different standards, and students are free to choose which university to attend. Naturally it is more costly to teach to a higher standard. The model determines the types of degrees present in the market and the corresponding allocation of students in the various degrees. The equilibrium picture emerging is like the one typically observed in reality, with costly "top" colleges setting high standards and enrolling best students, and less expensive colleges with lower standards and more average students.

After establishing existence of equilibrium, the model is used to interpret some facts relating to the observed allocations of students into colleges, and to discuss subsidies to education. Equilibrium structure makes clear that "low-rank" universities are there because they achieve an important goal: to give the non-excellent students adequate education. In essence, in equilibrium students with differing learning ability buy different goods at different prices. Regarding policy, the general conclusion seems to be that, not counting education externalities and policy costs, subsidies decrease welfare, thus they may be beneficial only if externality effects are large enough to compensate costs and reduction in welfare. What happens is that drawing more people into the bad schools makes them less desirable to their better students who then move to the middle schools and drag down the quality there and welfare with it. This seems to us a real effect that one should worry about.

In the context of the economics literature on education, the distinguishing feature of our paper lies in the source of equilibrium stratification. In the existing literature the emergence and desirability of stratification by ability generally depend on the strength of peer effects.[2] In our equilibrium stratification arises for a different reason, namely that more able students generate higher value from higher standards. Thus the relevance of our model hinges on the plausibility of the assumption that university's choice is simply to set an educational standard. In our understanding, the main existing alternative reflects the peer-effect view just discussed, and boils

the fierce debate on affirmative action. The book by Bowen-Bok (1998) is an example. Espenshade-Walton (2009) criticize Bowen and Bok's empirical analysis, and their conclusion is more agnostic.

[2]This is also the case in more general social interaction contexts, see for example Benabou (1996a,b).

down to assuming that universities maximize the quality of their student body, as in the fundamental work of Epple and Romano (in the papers we cite and several others), as well as the more recent MacLeod-Urquiola (2011) and Fu (2011).[3] Under this assumption the source of the observed heterogeneity in quality of student body across colleges is an exogenous difference among universities (in productivity or external fundings) or among student preferences regarding peers. The present paper on the other hand puts teaching standards at center stage, in their relation to learning ability.

The obvious link between teaching standards and learning ability - that better students respond better to harder material - has not been much stressed in the literature on the value of education in the labor market, although the separate elements have always been there and empirical evidence on the link is available (see for instance Light-Strayer (2000)). Student ability is of course an issue since at least Griliches-Mason (1972), as extensively surveyed in the Handbook articles Card (1999) and Heckman-Lochner-Todd (2007). Various aspects of school quality have also been under scrutiny for a long time (mainly in high school models): for example in Card-Krueger (1996) as a factor increasing the marginal value of time spent in education, or in Epple-Romano (1998) where, as we mentioned, quality is essentially identified with average ability of student body. And the role of teachers quality has been uncovered more recently, again in the context of schools rather than higher education, starting with the influential paper Rivkin et al (2005); the line of research is still active, see for example Tincani (2012). Note however that in the university context we have in mind the teaching situation is different than in schools, because the choice of material to teach is more flexible and significant.

2 University Policy and University Choice

2.1 The model

Students may attend universities where they study and are granted degrees. The quality of a student is denoted by $0 \leq q \leq 1$ and is publicly known. This represents native ability, plus human capital and the stock of knowledge accumulated at end of

[3] A different model is proposed by DeFraja-Valbonesi (2012) who postulate that universities maximize the amount of research they carry out. Their resulting picture, quite different than ours, has universities which carry out more research also enrolling more students and charging lower tuition fees.

high school. In accordance with this interpretation, students with higher endowments will be able to learn more difficult material. The density of students of different qualities is given by a continuous and positive density function $f(q)$. We abstract from student effort.

There are three types of universities: Pass (P), Middle (M) and Honors (H). Each university $i = P, M, H$ sets an educational quality standard $Q_i \in [0, 1]$. The standard Q_i represents the difficulty of the material that is taught. Our goal is to study which students choose to attend which universities.

The different types of universities are distinguished by their choice of Q. An Honors university "teaches to the top", that is, it sets Q at the highest possible value, $Q_H = 1$. This is the best which can be done to favor good students. A Middle university "teaches to the middle." That is, if students between $[q_M, q_H]$ are enrolled in the university, it sets a standard $Q(q_M, q_H)$ that is intermediate in the sense that $q_M \leq Q(q_M, q_H) \leq q_H$. We assume that $Q(q_M, q_H)$ is continuous, strictly increasing in q_M and q_H for $q_M, q_H < 1$. It is a generalization of the average of enrolled students, which satisfies these properties. A Pass university "teaches to the bottom" that is, it sets Q equal to least q of students enrolled in that university; so a Pass university favors the weakest.

Notice that the behavior of the universities is specified exogenously. One can view this as an evolutionary theory of universities: a given "type" of university follows a given policy, and it is then either successful at drawing students or not.

An important example motivating this model is the public university system in California. This has three tiers: at the top the University of California system corresponds to our H, the California State University system corresponds to our M and the junior college system corresponds to our P. Notice that while each system has many independent campuses within each system admissions standards and the educational quality standards are similar.

A student receives the market value of her education less the cost of obtaining the degree. The value of education depends on student quality q and on the standard Q to which she is taught; the cost depends on the standard of teaching Q. Hence the net market value of a student of quality q taught to standard Q is denoted $v(q, Q)$, which depends on both. We will measure Q so that student q's value is maximized at $Q = q$: $v(q, q) \geq v(q, Q)$ for all Q.

This value function embeds the "mismatch" hypothesis mentioned in connection to the affirmative action debate, but of course is more general. Affirmative action aside, the fact that achievement depends on the matching between student learning potential and college educational standard is found for example in Light-Strayer (2000) where achievement is measured by probability of graduation.

We are assuming that universities have a constant marginal cost per student of providing education that depends only on the standard set by that university. However, Honors universities have a capacity constraint. This may represent scarcity of quality faculty needed to teach the deepest material. We can represent this constraint by a cutoff q_H^{lb} representing the quality level such that the mass of students between $[q_H^{lb}, 1]$ equals the capacity of the Honors universities. Universities price their services at marginal cost. When the capacity constraint binds on the Honors universities, this means that the competitive rents from the scarce capacity is passed on to the students and not the universities or their faculty. Consequently costs are borne by the students who choose to attend those universities. The marginal cost pricing on the part of universities is supported by Epple-Romano-Sieg (2006).

A student of quality q applies to a university i which gives highest value $v(q, Q_i)$, or chooses not to go to university if this is negative for all i. Pass and Middle universities admit all students who apply. Honors universities admit the best students that apply if applications exceed their capacity. Any students who are rejected are assumed then to apply to their second choice - where they are admitted.

Definition. An equilibrium is an allocation of students to university types, that is a partition of $[0, 1]$ consistent with the application and admission procedure.

In effect we assume that the universities and students both know student quality q. Universities also know the cutoffs and the distribution of student qualities, so can determine their teaching standard. Students know these standards, but do not know the actual cutoffs - so when the capacity constraint binds on the Honors universities there will be students who apply - because they prefer the Honors university - but are turned away - because they are below the cutoff induced by the capacity constraint.

Coming back to value, the function v is assumed to be continuous. For a student of quality q there is a target level of quality that maximizes that student's value. If Q is too high the student will not be able to grasp the material. If Q is too low the student will learn little of value. As we said, we measure Q so that student q's value is maximized strictly at $Q = q$: more precisely, $v(q, q) \geq v(q, Q)$ for all Q, and $v(q, Q)$ is strictly increasing in Q for $Q < q$ and strictly decreasing for $Q > q$. We also assume that $v(q, Q)$ is strictly increasing in q, so that a better student has a higher market value for any given educational standard.

Observe that $v(q,q)$ is strictly increasing - that is, it is worthwhile to provide a better education to better students. This follows from the fact that $v(q,Q)$ is strictly increasing in q and that $v(q,q) \geq v(q,Q)$. We assume that $v(0,0) < 0$ so that the worst conceivable student cannot profit from college education, and that $v(1,1) > 0$ so that it is profitable to provide the highest standard of education to a suitably talented student. It follows that there is a unique value $\underline{q}$ that satisfies $v(\underline{q},\underline{q}) = 0$. This in turn implies that no student with $q < \underline{q}$ will attend a university, while any student with a higher value of will attend some university. Notice that since $v(1,1) > 0$ it must be that $\underline{q} < 1$.

As we noted, $v(q,q)$ is strictly increasing. Moreover $v(q,q) > v(q,Q)$ for $Q \neq q$, which is to say, the function $v(q,Q) - v(q,q)$ has a maximum with respect to q at $q = Q$. We strengthen this slightly to assume that $v(q,Q) - v(q,q)$ is in fact single-peaked.

Assumption 1. *For fixed Q, the function $v(q,Q) - v(q,q)$ as a function of q is single-peaked.*

This says that as q approaches the value for which Q is best the loss from choosing an inferior q diminishes. We shall maintain this assumption in what follows.

2.2 Comments on the model

1. Our assumption that $v(q,Q)$ peaks exactly at $Q = q$ is not a substantial restriction. It is equivalent to having a v peaking at any $Q^*(q) \neq q$, from which it can be obtained by transforming the Q^* function; of course the transformation affects averaging, and this is why we leave the function $Q(q_M, q_H)$ more general than the average $\int_{q_M}^{q_H} qf(q)dq / \int_{q_M}^{q_H} f(q)dq$.

Empirical investigations about the actual location of the maximum Q given q are represented by the regression question of whether given ability the selectivity of the college attended helps in the labor market. Answers are mixed, see for example Dale-Krueger (2002, 2011), Brewer-Eide-Ehrenberg (1999) or Li et al. (2012) on Chinese data.

2. We have assumed that marginal cost is constant in the number of students enrolled (for each given educational standard) up to the capacity constraint. In this linear technology the college is the marginal unit in the production of educational service.

3. We abstract from other complications which we briefly mention here: (a) uncertainty about q and v; (b) student effort; (c) student wealth and credit market imperfections; (d) for given q, Q other idiosyncratic factors influencing v, for example the presence of a family firm.

2.3 Characterization of equilibrium

The first implication of Assumption 1 is that better students profit more from a given difference in standards:

Lemma 1. *For given $Q' > Q$ the function $v(q, Q') - v(q, Q)$ is strictly increasing for $q \in [Q, Q']$.*

Proof. Writing $v(q, Q') - v(q, Q) = v(q, q) - v(q, Q) - [v(q, q) - v(q, Q')]$ the conclusion is direct from the assumption. □

Incidentally, it is clear from the proof that the converse does not hold. We also have

Lemma 2. *If student q is indifferent between a pair Q_i, Q_j with $Q_i < Q_j$, then any student $q' < q$ will prefer Q_i, and any $q' > q$ will prefer Q_j.*

Proof. The hypothesis $v(q, Q_j) = v(q, Q_i)$ implies (given strict maximum at $Q = q$) that $Q_i < q < Q_j$. Now if $q' \in [Q_i, Q_j]$ the assertion follows directly from Lemma 1; if $q' \notin [Q_i, Q_j]$ it follows from single-peakedness of $v(q, Q)$ as a function of Q. □

In principle equilibrium partitions of students into universities may be arbitrarily complex, but under our maintained assumptions the partition of interest, with fully populated degrees, is quite simple:

Proposition 1. *Equilibrium with students in all three types of degree is characterized by cutoffs $\underline{q} < q_M < \hat{q}_H < 1$ such that students q below $\underline{q}$ do not attend university, those between $\underline{q}$ and q_M attend Pass, those between q_M and $q_H = \max\{q_H^{lb}, \hat{q}_H\}$ attend Middle, and those above q_H attend Honors. Student q_M is indifferent between Middle and Pass, student $\hat{q}_H$ between Middle and Honors. Conversely if there are two cutoffs $q_M, \hat{q}_H$ with $\underline{q} < q_M < \hat{q}_H < 1$ that satisfy these properties then the partition they determine (with $q_H = \max\{q_H^{lb}, \hat{q}_H\}$) constitutes an equilibrium with fully populated degrees.*

Proof. Assume equilibrium with students in all three types of degree. That students with $q < \underline{q}$ do not attend university follows from the fact that for such q it is $v(q, Q) < 0$ for all $Q \in [\underline{q}, 1]$. And we know that the least in Pass is the $\underline{q}$ such that $v(\underline{q}, \underline{q}) = 0$. Letting Q_M be the standard at Middle, clearly $Q_M \in [\underline{q}, 1]$; moreover, if Q_M is at a boundary then Middle is equal to one of the other degrees hence effectively empty, so we can assume $\underline{q} < Q_M < 1$.

Consider the function $v(q, \underline{q}) - v(q, Q_M)$, the gain from attending Pass over Middle. It is positive at $q = \underline{q}$ and negative at $q = Q_M$, so there is a q_M with $\underline{q} < q_M < Q_M$ where it vanishes: $v(q_M, \underline{q}) - v(q_M, Q_M) = 0$. Student q_M is indifferent between $\underline{q}$ and $Q_M > \underline{q}$, so by Lemma 2 all students below q_M will prefer Pass over Middle and all those above q_M prefer Middle. Consider next the gain of Middle over Honors, $v(q, Q_M) - v(q, 1)$. This is positive for all $q \in [q_M, Q_M]$ and negative at $q = 1$, so there is a student $\hat{q}_H$ with $Q_M < \hat{q}_H < 1$ who is indifferent between Middle and Honors: $v(\hat{q}_H, Q_M) - v(\hat{q}_H, 1) = 0$. Lemma 2 then implies that all $q < \hat{q}^H$ prefer Middle to Honors and all $q > \hat{q}^H$ prefer Honors. We can therefore conclude that all q with $q_M < q < \hat{q}_H$ prefer Middle over both Pass and Honors; by single-peakedness of $v(q, \cdot)$ it is also the case that all $q < q_M$ will a fortiori prefer Pass also to Honors, and that all $q > \hat{q}_H$ similarly prefer Honors also to Pass. Hence $q_H = \max\{\hat{q}_H, q_H^{lb}\}$.

Conversely, assuming cutoffs as in statement the preference orderings of the different degrees are established with the same arguments as in the first part of the proof, using again single-peakedness and Lemma 2. $\square$

The capacity constraint is binding if $\hat{q}_H < q_H^{lb}$. In this case the least Honors student q_H is q_H^{lb}, and strictly prefers Honors to Middle. If on the other hand $\hat{q}_H \geq q_H^{lb}$ then $q_H = \hat{q}_H$ and equilibrium is unconstrained.

2.4 Existence of equilibrium

The cutoffs $q_M, \hat{q}_H$ defined in Proposition 1 are computed so as to be indifferent between Middle and the adjacent degree. But note that if we start with arbitrary $q_M < \hat{q}_H$ and then compute $Q(q_M, q_H)$ it is not necessarily the case that those values satisfy the required indifferences. As usual, equilibrium is a fixed point of a suitably defined map. Instead of looking for a fixed point we will equivalently work with a suitably defined vector field. To this we turn.

We define two gain functions, defined for arbitrary q_M, q_H with $\underline{q} \leq q_M \leq q_H \leq 1$. One is the gain to attending a Pass university over a Middle university for student

q_M:

$$G_P(q_M, q_H) = v(q_M, \underline{q}) - v(q_M, Q(q_M, q_H)).$$

The second is the gain to attending a Middle university over a Honors university for student q_H:

$$G_M(q_M, q_H) = v(q_H, Q(q_M, q_H)) - v(q_H, 1).$$

Recall that $q_H = \max\{q_H^{lb}, \hat{q}_H\}$. Proposition 1 says that equilibrium with fully populated degrees is either a zero of the vector field (G_M, G_P) with $\underline{q} < q_M < q_H = \hat{q}_H < 1$, in which case it is not capacity constrained, or a (q_M, q_H) such that $q_H = \underline{q_H}$, $G_P(q_M, q_H) = 0$ and $G_M(q_M, q_H) < 0$. Geometrically this latter condition says that the vector field is outward normal to the feasible set of q_M, q_H on the boundary where $q_H = q_H^{lb}$.

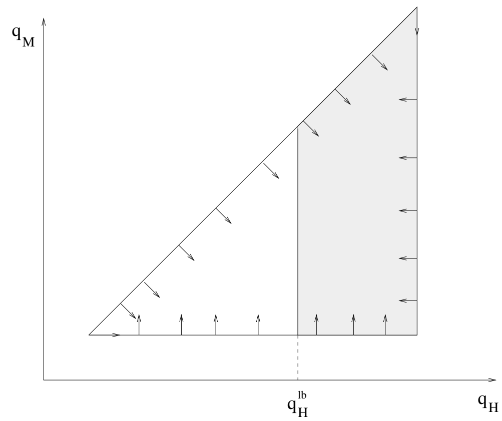
Theorem 1. *An equilibrium with fully populated degrees exists.*

To prove this we will use the following useful corollary of the Brouwer fixed point theorem.

Lemma 3. *A continuous vector field on a compact convex subset X with non-empty interior of a finite dimensional vector space either has a zero or there is a boundary point where the vector field is normal to X.*

Proof. Suppose the vector field is $\phi(x)$. Consider the (continuous) map $M(x) = x + \phi(x)$, and the map $P(x)$ that project points to the closest point in X. Since X is compact and convex, the projection map is well-defined and continuous. Hence $P \circ M$ is a continuous map from X to itself and so by the Brouwer fixed point theorem has a fixed point. An interior fixed point is a zero of the vector field; a boundary fixed point is either a zero of the vector field, or the projection of $x + \phi(x)$ is the same as x meaning that $\phi(x)$ is normal to X. □

Proof of Theorem. We find it easier to visualize triangles below the 45-degree diagonal, so we work on the plane (q_H, q_M), with q_H on the horizontal axis. Consider the square with sides $q_H, q_M \in [\underline{q}, 1]$, and the triangle below its 45-degree diagonal, where $q_H \geq q_M$, as in the figure below. To apply Lemma 3 we just have to check that the vector field (G_M, G_P) can only have an outward normal along the vertical boundary at $q_H = q_H^{lb}$.



1. Start with the line $q_M = q_H = q$, where $Q(q,q) = q$. By Assumption 1 the function $G_M(q,q) = v(q,q) - v(q,1)$ is strictly positive and decreasing for all $q < 1$, and it reaches zero at 1. On the other hand, consider the function $G_P(q,q) = v(q,\underline{q}) - v(q,q)$. First notice that $G_P(\underline{q},\underline{q}) = 0$. It is $G_P(q,q) = -[v(q,q) - v(q,\underline{q})]$ so Assumption 1 implies that in this interval it increases to zero (which it reaches at $\underline{q}$) then decreases. In conclusion, on the diagonal the vector field points south-east for all $\underline{q} < q < 1$, while on the lower-left corner it points rightwards, and on the upper-right corner points downwards.

2. On the horizontal boundary at $q_M = \underline{q}$ the vector field points to the interior of the triangle if $G_P(\underline{q}, q_H)$ is positive for $q_H > \underline{q}$. Set $Q = Q(\underline{q}, q_H)$. Since $q_H > \underline{q}$ it is $Q > \underline{q}$, and on the other hand we know that $v(\underline{q},\underline{q}) = 0 > v(\underline{q}, Q)$ for all $Q \neq \underline{q}$; therefore $G_P(\underline{q}, q_H) = v(\underline{q},\underline{q}) - v(\underline{q}, Q) > 0$.

3. On the vertical boundary at $q_H = 1$ G_M is negative for $q_M < 1$, because for such values $Q(q_M, q_H) < 1$, so $G_M(q_M, 1) = v(1, Q) - v(1,1) < 0$. Thus again it points inwards.

The conclusion is that the vector field can only have an outward normal along the vertical boundary at $q_H = q_H^{lb}$, as was to be shown. □

2.5 Uniqueness of Equilibrium When the Constraint Binds

Observe that we have not shown that equilibrium is unique, and in fact there may be multiple equilibria. The intuition for two different equilibria is that, with Q_M denoting the standard of Middle, low Q_M makes Honors attractive for more students, and this in turn lowers Q_M; and conversely high Q_M makes Middle attractive for good students, and this in turn drives Q_M upward.

To ensure uniqueness, at least in the relevant case where the capacity constraint binds, we can strengthen the property of v described in Lemma 1 that for given $Q' > Q$ the function $v(q, Q') - v(q, Q)$ is strictly increasing for $q \in [Q, Q']$, as stated in the next Lemma.

Proposition 2. *Suppose the capacity constraint binds. Then equilibrium is unique if the function $v(q, Q(q, q_H^{lb})) - v(q, \underline{q})$ is strictly increasing in q.*

Proof. In equilibrium student q_M is indifferent between Pass and Middle, which when constraint binds amounts to the condition $v(q_M, Q(q_M, q_H^{lb})) - v(q_M, \underline{q}) = 0$. On the other hand we have $v(\underline{q}, Q(\underline{q}, q_H^{lb})) - v(\underline{q}, \underline{q}) < 0$ because $Q(\underline{q}, q_H^{lb}) \neq \underline{q}$, and if the monotonicity condition in the Lemma holds the function $v(q, Q(q, q_H^{lb})) - v(q, \underline{q})$ cannot have more than one zero. □

Note that $Q(q, q_H^{lb}) > q > \underline{q}$ so the condition is indeed a strengthening of the property in Lemma 1. The new element here is that as q goes up the higher Q', now $Q(q, q_H^{lb})$, moves upwards too. To see what the monotonicity entails we may look at the derivative (subscripts for partials)

$$\frac{\partial}{\partial q}[v(q, Q(q, q_H^{lb})) - v(q, \underline{q})] = [v_1(q, Q(q, q_H^{lb})) - v_1(q, \underline{q})] + v_2(q, Q(q, q_H^{lb})) \cdot Q_1(q, q_H^{lb});$$

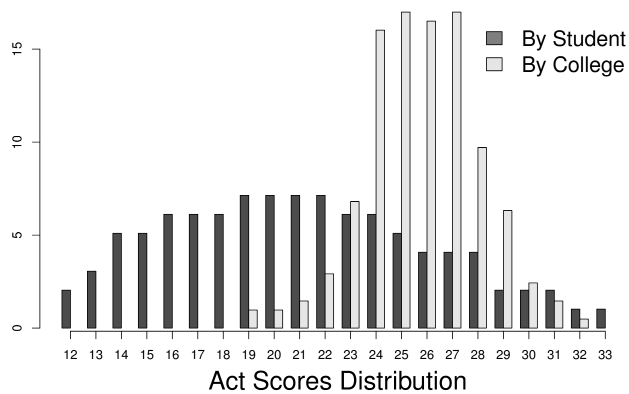
the term in brackets is positive by Lemma 1, and Q_1 is positive by assumption; on the other hand since $Q(q, q_H^{lb}) > q$ it is $v_2 < 0$. So a condition ensuring that the derivative is positive is that v_2 be not too large, which says that student q does not lose too much when teaching standard moves towards higher levels than would be optimal for her.

3 Basic Facts Related to the Model

We have already observed that the organization of the State supported university system in California bears a fairly strong resemblance to our equilibrium. We now look at a couple more specific pieces of evidence related to the model we have presented.

3.1 Distribution of ACT test scores

One measure of student ability is given by ACT scores, whose distribution for the USA in recent years is available.[4] A fact about these scores is that the distribution of scores over students is relatively normally distributed (the test is normalized to make this true) - and in particular has a central peak. By contrast the distribution of the average ACT scores for colleges is quite flat, in particular flatter than the distribution over students in the relevant range.[5] See the figure below.



If universities are very specialized, so that - unlike our theory - there are many universities of homogeneous size, each matched with students of similar quality then the distribution of average scores for colleges should be similar to that over students. In our theory, however, with a small number (three) of types of universities, the cutoffs occur independent of the distribution of student qualities. Start with a distribution of student qualities so that the distribution of college qualities is similar. Suppose that the distribution of student qualities becomes more concentrated. Since the cutoffs do not change and the number of colleges of each type does not change (although their enrollment does), the distribution of college qualities remains unchanged, hence flatter that the distribution of student qualities. In our theory the distribution of average scores over colleges will flatten sufficiently steeply peaked distribution of student scores, in accordance with the observed data.

[4]See `actstudent.org/scores/norms1.html`. Data are for 2010-2012 high school graduates.

[5]We derived this distribution from data at `stateuniversity.com/rank/act_75pctl_rank/25`. It is shifted to the right compared to the distribution by students, because that includes students not going to university.

3.2 Changing selectivity in American colleges

Hoxby (2009) finds that in the period 1960-2007 in the USA selectivity has increased for the top colleges but decreased for the rest. As we shall now see our model accommodates the first fact but not the second. Hoxby argues that the phenomenon is produced by a re-sorting of students due to decreased mobility costs: students care less about the proximity of universities and more about quality. In terms of our model this says that $v(q, 1)$ shifts upwards. The reason is that mediocre universities have always been available nearby, but top universities are fewer and more distant. However with decreased mobility costs also the top colleges are effectively nearby; this lowers costs for the students aiming at Honors. This implies a lower $\hat{q}_H$, hence a higher ratio of applications over admissions - greater selectivity - at capacity constrained Honors colleges. The threshold q_M, hence the alleged selectivity of the other types of colleges, is however not affected by the demand shift.

4 Comparative Statics: Subsidizing Education

Equilibria in our model, constrained or not, is generally not efficient in the sense of maximizing the average value of v over the population of students. We now turn to examining the comparative statics of the equilibrium - which amounts to study the effects of perturbations of the value function v - and the consequences for welfare. We will focus on the case in which the capacity constraint is binding, $\hat{q}_H < q_H^{lb}$, as that is the empirically relevant case. So the least Honors student q_H will be fixed at q_H^{lb}, and the relevant threshold will be the least student q_M who attends Middle. Note that the larger the distance between the indifferent $\hat{q}_H$ and the capacity limit q_H^{lb} the more are the applicants to Honors relative to admissions. The uniqueness condition in Lemma 2 is assumed to hold.

Exogenous variations of value v represent education policy, and the question is how to increase welfare, in particular average v over student population, through policy interventions. Given $\underline{q}$ and q_H^{lb} welfare only depends on q_M, so the issue boils down to how q_M can be raised or lowered, as the case may require, through policy measures.

Remark. We will not analyze relaxing the capacity constraint (i.e. lowering q_H^{lb}) because we regard it as not feasible. Such a policy would raise value for students in $[\hat{q}_H, q_H^{lb}]$ affected by the shift; on the other hand it would lower the Middle standard so it would worsen the position of those students in $[\hat{q}_H, q_H^{lb}]$ who remain in Middle, and it would also lower the least enrolled there, with ambiguous effects on welfare.

We next write down the welfare measure as average v and look at conditions determining the sign of the change in q_M required to increase welfare, and then specify possible policies and see what they can achieve .

Since welfare as average v depends on q_M we denote it $W(q_M)$. Also the Middle standard $Q(q_M, q_H)$ will only depend on q_M, so it will be written as $Q(q_M)$. Students below $\underline{q}$ do not attend university so their value is fixed at zero. Then we have

$$W(q_M) = \int_{\underline{q}}^{q_M} v(q, \underline{q}) f(q) dq + \int_{q_M}^{\underline{q_H}} v(q, Q(q_M)) f(q) dq + \int_{\underline{q_H}}^{1} v(q, \bar{q}) f(q) dq$$

First and last terms do not depend on q_M. Computing the derivative and denoting by v_2 the partial derivative of $v(q, Q)$ with respect to Q we get

$$\begin{aligned} W'(q_M) &= [v(q_M, \underline{q}) - v(q_M, Q(q_M))] f(q_M) + \int_{q_M}^{\underline{q_H}} v_2(q, Q(q_M)) \cdot Q'(q_M) f(q) dq \\ &= Q'(q_M) \int_{q_M}^{\underline{q_H}} v_2(q, Q(q_M)) f(q) dq \end{aligned}$$

where the last expression holds at equilibrium using the fact that q_M is indifferent between Pass and Middle. Since $Q'(q_M) > 0$ the value of this derivative depends on the integral, and we can thus state for reference

Proposition 3. *Assume that capacity constraint is binding. Then $W'(q_M) > 0$ at equilibrium q_M iff*

$$\int_{q_M}^{\underline{q_H}} v_2(q, Q(q_M)) f(q) dq > 0.$$

Note that the integrand is negative from q_M up to $Q(q_M)$, then positive, for students below $Q(q_M)$ are worse off if $Q(q_M)$ gets larger while the opposite occurs for $q > Q(q_M)$. So re-writing the above inequality as

$$\int_{Q(q_M)}^{\underline{q_H}} v_2(q, Q(q_M)) f(q) dq > - \int_{q_M}^{Q(q_M)} v_2(q, Q(q_M)) f(q) dq$$

we see that W increases with q_M if the gain that a higher q_M brings to the Middle-attending students above $Q(q_M)$ more than offsets the loss which it causes to the students below $Q(q_M)$. To assess whether this condition is likely to hold observe that all students who would prefer Honors and attend Middle because of capacity constraints are on the right of $Q(q_M)$, and their preferred Q is the highest possible. If the mass of these students is heavy enough the gain to them of an increase in the standard $Q(q_M)$ becomes larger than the loss suffered by students on the left of $Q(q_M)$. In this case the condition under discussion holds so that to increase welfare one should raise q_M. Given observed strong selectivity of top colleges this would seem to be a rather typical case.

Remark. Of course equilibrium may also happen to be efficient. It is readily verified that if f is uniform, v_2 is linear and $Q(\cdot)$ is the average function then $W'(q_M) = 0$, so if W is concave equilibrium is efficient.

4.1 Policy

There are two types of policy interventions on $v(q, Q)$. One possibility is to intervene on student value, resulting in a function $v(q, Q) + \alpha\tilde{v}(q)$. Alternatively the intervention may be based on the type of University $v(q, Q) + \beta\tilde{v}(Q)$. The first kind of intervention is ineffective since it has no effect on the indifference conditions. Hence the the interesting policy - quite realistically in fact - is of the form $v(q, Q) + \beta\tilde{v}(Q)$, changing relative benefits of attending the different types of universities for all students alike.

A simple type of intervention can be represented by $\tilde{v}(Q) = \mathbf{1}_{Q_i}(Q)$ (the indicator function of Q_i), so that $v(q, Q) + \beta\tilde{v}(Q)$ means paying β to those attending Q_i for an $i \in P, M, H$ if $\beta > 0$, or taxing them if $\beta < 0$. This type of intervention is discussed next, for each of the three degree types, in each case denoting by $q_M(\beta)$ the equilibrium threshold corresponding to β, with $q_M(0)$ being the original value q_M.

1. We consider first a simple subsidy to education (i.e. you are paid a fixed amount to attend - could be a fee rebate or whatever) regardless of quality. This lowers $\underline{q}$ hence also lowers q_M by making the lower school less attractive to the better students. Overall enrollments go up and also at the middle school. The enrollments at the lower school is ambiguous. Quality of education declines as it is lower at both the lower and middle school, and the average quality of education declines. We are assuming welfare increases in q_M so this subsidy lowers welfare at some cost.

2. Consider next a simple subsidy just to the Pass school. This has the identical effect on $\underline{q}$ and a weaker possibly ambiguous effect on q_M since you have to go to the lower school to get the subsidy. Overall enrollments go up just as much; enrollments at the middle school go up less than with a simple subsidy (and perhaps even down); The quality of education declines just at much at the lower school, but goes down less at the middle school since q_M does not fall as much as with the simple subsidy. It seems possible here that the average quality of education goes up if the effect on the middle school is strong enough. This is less bad for welfare as it does not lower q_M as much and may in fact raise it.

3. Consider now the case of subsidizing Middle Degrees ($\beta > 0$; the case $\beta < 0$ is analogous). It has no effect on $\underline{q}$ but lowers q_M since you have to go to the middle school to get the subsidy. No overall effect on enrollments; enrollments drop at the lower school and go up at the middle school. Educational quality is unchanged at the lower school, and goes down at the middle school. The overall effect on the average is ambiguous, since there are more middle school students getting a poorer education. The effect of this measure on welfare is negative as it lowers q_M.

4. Lastly consider subsidizing only Honors Degrees. Since there are more students wanting to attend Honors than those who can because of capacity constraints, marginal intervention on Honors just makes rationing more severe if $\beta > 0$ or less severe if $\beta < 0$. No thresholds are changed in equilibrium, therefore such a policy is ineffective.

4.1.1 Summing up

We have seen that it is ineffective to intervene by modifying incentives to attendance to Honors. As regards to Pass, assuming the effect on q_M of an intervention on Pass is negligible evaluation of a policy subsidizing attendance to Pass universities amounts to evaluating the positive externality of the extra students attending university, which is in practice largely a political decision.

The debatable issue concerns financing Middle universities. For marginal perturbations (small β) the question is whether the condition in Proposition 3 holds or not, and we have argued that it may indeed do. If this is the case then it would not be a good idea to subsidize attendance to Middle at the margin because this would lower $Q(q_M)$ (the average quality at Middle) and welfare with it, and to increase welfare the Middle degrees should be instead penalized relative to the other two. The intuition for discouraging participation to Middle degrees is that this measure displaces students at the bottom end of Middle who were unconstrained originally, while benefiting students in the upper tail of Middle who were constrained originally (by Honors capacity) hence may have greater advantage from the increased Middle standard.

The general conclusion seems to be that not counting education externalities and policy costs subsidies decrease welfare, and therefore may be beneficial only if externality effects are large enough to compensate costs and reduction in welfare. Essentially, drawing more people into the bad schools makes them less desirable to their better students who move to the middle schools and drag down the quality there and welfare with it. This seems to us a real effect that one should worry about.

5 Concluding Remarks

We do not subscribe to the view that a university's quality is determined by that of its student body, and have proposed instead a model that shifts focus more on teachers and educational standards, where good students are driven to colleges which set high standards because they have better learning potential hence extract more value from the useful but hard material which is taught in those colleges.

The model stands well against observed distribution of tests scores by student and by college, and as we observe it may also have some bearing on grades and evaluation of teaching (measured as alignment of teachers to university's objectives).

Stratification by student ability then emerges in equilibrium as a natural consequence of the student-college quality matching, and it effectively achieves the purpose of providing adequate education to the non-excellent students.

The model provides a coherent, equilibrium foundation for the arguments and empirical studies competing in the debate on affirmative action in university, which are all centered upon the idea of educational value being based on the relation between student skill and university standard (in our terms a v function).

Taking average student market value as a measure of welfare, performance of equilibrium in terms of welfare depends on the cutoffs which determine students allocation in the various types of degrees, hence the effects of education policy measures depend on how they affect these cutoffs. The general conclusion about policy seems to be that, not counting education externalities and policy costs, subsidies to education decrease welfare. Thus they may be beneficial only if externality effects are large enough to compensate costs and reduction in welfare. Essentially, drawing more people into the bad schools makes them less desirable to their better students who move to the middle schools and drag down the quality there and welfare with it. This seems to us a real effect that one should worry about.

References

Benabou, Roland (1996): Equity and efficiency in human capital investment: the local connection, *Review of Economic Studies* **63**, pp. 237-264

Benabou, Roland (1996): Heterogeneity, stratification and growth: macroeconomic implications of the community structure and school finance, *American Economic Review* **86**, pp. 584-609

Bowen, William G., and Derek Bok (1998): *The Shape of the River: Long-Term Consequences of Considering Race in College and University Admissions*, Princeton: Princeton University Press

Brewer, Dominic J., Eric R. Eide and Ronald G. Ehrenberg (1999): Does It Pay to Attend an Elite Private College? Cross-Cohort Evidence on the Effects of College Type on Earnings, *The Journal of Human Resources* **34**, pp. 104-123

Card, David (1999): The Causal Effect of Education on Earnings, in *Handbook of Labor Economics*, Edited by O. Ashenfelter and D. Card, *Elsevier Science*

Card, David and Alan B. Krueger (1996): Labor Market Effects of School Quality: Theory and Evidence, NBER Working Paper

Dale, Stacy B. and Alan B. Krueger (2002): Estimating the Payoff to Attending a More Selective College: An Application of Selection on Observables and Unobservables, *The Quarterly Journal of Economics* **117**, pp. 1491-1527

Dale, Stacy B. and Alan B. Krueger (2011): Estimating the Return to College Selectivity Over the Career Using Administrative Earnings Data, NBER WP 17159

De Fraja, Giovanni and Paola Valbonesi (2012): The Design of the University System, *Journal of Public Economics* **96**, pp. 317-330

Espenshade, Thomas J. and Alexandria Walton Radford (2009): *No Longer Separate, Not Yet Equal,* Princeton: Princeton University Press

Epple, Dennis and Richard Romano (1998): Competition Between Private and Public Schools, Vouchers, and Peer-Group Effects, *American Economic Review* **62**, pp. 33-62

Epple, Dennis and Richard Romano (2011): Peer Effects in Education: A Survey of the Theory and Evidence, in *Handbook of Social Economics Volume 1*, ed. Jess Benhabib, Alberto Bisin, and Matthew O Jackson, pp. 1053-1163

Epple, Dennis, Richard Romano and Holger Sieg (2006): Admission, Tuition, and Financial Aid Policies in the Market for Higher Education, *Econometrica* **74**, pp. 885-928

Fu, Chao (2011): Equilibrium Tuition, Applications, Admissions and Enrollment in the College Market, mimeo University of Wisconsin

Griliches, Zvi and William M. Mason (1972): Education, Income, and Ability, *Journal of Political Economy* **80**, pp. S74-S103

Heckman, James J., Lance Lochner and Petra E. Todd (2007): Earnings Functions, Rates of Return and Treatment Effects: the Mincer Equation and Beyond, in *Handbook of the Economics of Education, Volume 1,* Elsevier

Hoxby, Caroline M. (2009): The Changing Selectivity of American Colleges, *Journal of Economic Perspectives* **23**, pp. 95-118

Hunt, Shane (1963): "Income Determinants for College Graduates and the Return to Educational Investment, Ph.D. thesis, Yale University

Kling, Jeffrey R., Jeffrey B. Liebman and Lawrence F. Katz (2007): Experimental Analysis of Neighborhood Effects, *Econometrica* **75**, pp. 83-119

Li, Hongbin, Lingsheng Meng, Xinzheng Shi and Binzhen Wu (2012): Does attending elite colleges pay in China? *Journal of Comparative Economics* **40**, pp. 78-88

Light, Audrey and Wayne Strayer (2000): Determinants of College Completion: School Quality or Student Ability? *The Journal of Human Resources* **35**, pp. 299-332

MacLeod, W. Bentley and Miguel Urquiola (2011): Anti-Lemons: School Reputation, Relative Diversity, and Educational Quality, mimeo Columbia University and NBER

Pozzo, Michela and Vincenzo Scoppo (2012): The Good, The Bad, And The Ugly: Teaching Evaluations, Beauty And Abilities, mimeo

Rivkin, Steven G., Eric A. Hanushek and John F. Kain (2005): Teachers, Schools, and Academic Achievement, *Econometrica* **73**, pp. 417-458

Rojstaczer, Stuart and Christopher Healy (2012): Where A Is Ordinary: The Evolution of American College and University Grading, 1940–2009, *Teachers College Record* **114**

Sander, Richard H. (2004): A Systemic Analysis of Affirmative Action in American Law Schools, *Stanford Law Review* **57**, pp. 367-483

Sander, Richard H. (2011): Class in American Legal education, *Denver University Law Review* **88**, pp. 631-682

Sander, Richard and Stuart Taylor Jr. (2012): *Mismatch: How Affirmative Action Hurts Students It's Intended to Help, and Why Universities Won't Admit It*, Basic Books

Tincani, Michela (2012): Teacher Labor Markets, School Vouchers and Student Cognitive Achievement: Evidence from Chile, mimeo University of Pennsylvania